QCD corrections to the neutralino decay to an antisbottom and a bottom quark within MSSM

DIPLOMOVÁ PRÁCA

Hana Hluchá

UNIVERZITA KOMENSKÉHO V BRATISLAVE
FAKULTA MATEMATIKY, FYZIKY A INFORMATIKY
KATEDRA TEORETICKEJ FYZIKY A DIDAKTIKY FYZIKY

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Vedúci záverečnej práce
Mgr. Karol Kovařík, PhD

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Hana Hluchá

COMENIUS UNIVERSITY IN BRATISLAVA
FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS
DEPARTMENT OF THEORETICAL PHYSICS
AND PHYSICS DIDACTICS

field of study
PHYSICS

Supervisor
Mgr. Karol Kovařík, PhD

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I honestly declare that I have written the submitted master thesis by myself and I have used only the literature mentioned in the bibliography.

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Hana Hluchá

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Abstrakt

V mojej diplomovej práci odvádzam lagranžián pre teóriu MSSM vo formalizme superpriestoru a superpolí. Mojou snahou je, aby výsledná notácia čo najviac súhlasila s notáciou, ktorá sa používa na Inštitúte Fyziky Vysokých Energií vo Viedni. Počítam šírku rozpadu pre rozpad neutralína na antisbottom a bottom quark na jednoslučkovej úrovni s uvážením iba QCD korekcií, pričom používam Feynmanové pravidlá, ktoré vyplývajú z odvodeného lagranžiánu. V závere práce uvádzam grafy, na ktorých šírky rozpadu a hmotnosti častíc závisia od MSSM parametrov.

Diplomová práca je rozvrhnutá nasledovným spôsobom:

V prvej kapitole podávam stručný úvod do teórie supersymetrického modelu.

V druhej a tretej kapitole je načrtnuté odvodenie minimálneho supersymetrického lagranžiánu pre všeobecné superpolia.

V štvrtej kapitole dosádzam konkrétne superpolia, ktorých časticový obsah pozostáva z polí známych zo Štandardného Modelu ako aj z nových predpovedaných polí, ktoré ešte len čakajú na svoj objav na budúcich a možno už aj súčasných urýchľovačoch. Ďalej odvádzam vertexy, ktoré sú dôležité pri výpočte šírky rozpadu.

V piatej kapitole sa venujem renormalizácii MSSM. Zaoberám sa hlavne renormalizáciou sfermiónových ako aj fermiónových polí.

Posledná, šiesta kapitola obsahuje výpočet šírky rozpadu na stromovej ako aj na jednoslučkovej úrovni. Na konci tejto kapitoly uvádzam výsledné grafy.

V dodatku A, na konci práce, uvádzam prehľad vzťahov, ktoré sú dôležité pri práci so spinormi a Grassmanovými číslami. Zároveň definujem notáciu, ktorú používam.
V dodatku B sú uvedené ručne zrátané generické diagramy.

kľúčové slová: teória MSSM, formalizmus superpriestoru, rozpad neutralína, QCD korekcie

Abstract

In my master thesis I derive the lagrangian for the MSSM theory using the formalism of superspace and superfields. I try to be as close to the notation that is used at the Institute for High Energy Physics in Vienna as possible. I calculate the decay width for the neutralino decay to an antisbottom and a bottom quark at a one loop level considering only the QCD corrections using Feynman rules coming from the derived lagrangian. At the end of my thesis I present graphs where the decay widths and particle masses depend on various MSSM parameters.

The thesis is divided into the following parts:

In the first chapter I shortly introduce the theory of a supersymmetric model.

In the second and third chapter I present the derivation of the minimal supersymmetric lagrangian for the general superfields.

In the fourth chapter I put concrete superfields into the lagranian those particle content comprise of the fields known from the Standard Model as well as from the new predicted fields which are yet to be found on the future and maybe on the present-day colliders. I also derive the vertices important for the neutralino decay.

The fifth chapter deals with the renormalization of the MSSM. I focus mainly on the renormalization of the fermion as well as of the sfermion fields.

The last sixth chapter includes the calculation of the tree and one loop level decay width. At the end of the chapter I present the resulting graphs.

In the appendix A I present the summary of the identities which are essential to the work with spinors and Grassman numbers. I define the notation at the same time.
The appendix B involves the generic diagrams calculated by hand.

keywords: MSSM theory, superspace formalism, neutralino decay, QCD corrections

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Chapter 1

Introduction

The Standard Model of elementary particle physics (SM) is a remarkably successful theory of the known particles and their electroweak and strong forces. Although the SM correctly describes all known microphysical nongravitational phenomena, there are a number of theoretical and phenomenological issues that the SM fails to address adequately, whereas the MSSM provides explanations [1].

- *Hierarchy problem*
  Phenomenologically the mass of the Higgs boson associated with the electroweak symmetry breaking must be in the electroweak range. However, radiative corrections to the Higgs mass are quadratically dependent on the UV cutoff Λ, since the masses of the fundamental scalar fields are not protected by chiral or gauge symmetries. The "natural" value of the Higgs mass is therefore of $O(\Lambda)$ rather than $O(100 GeV)$, which leads to a destabilization of the hierarchy of the mass scales in the SM.
  The MSSM introduces new particles into the theory that couple to the Higgs and appear in the loop. These particles cancel the quadratic divergence and thereby solve the hierarchy problem.

- *Gauge coupling unification*
  In the contrast to the SM, the MSSM allows for the gauge coupling unification. The extrapolation of the low energy values of the gauge couplings using renormalization group equations and the MSSM particle content shows that the gauge coupling unify at the scale 3×10^{16} GeV. This quality lends credence to the picture of grand unified theories (GUTs) and certain string theories. Precise measurements of the law energy values of the gauge coupling demonstrated that the SM cannot describe gauge coupling unification.

- *Electroweak symmetry breaking (EWSB)*
  In the SM, electroweak symmetry breaking is parametrized by the Higgs boson h and its potential $V(h)$. However, the Higgs sector is not constrained by any symmetry principles, the negativeness of the parameter μ^2 is put into the theory by hand.
  The MSSM provides an explanation of the origin of EWSB.

- *Dark matter*
  In supersymmetric theories, the lightest superpartner (LSP) can be stable. This

particle provides a very good candidate for the cold dark matter.

Supersymmetry algebra

The theorem of Coleman and Mandula [2] demonstrates that the most general group of symmetries of the S-matrix is locally isomorphic to the direct product of a compact symmetry group and the Poincaré group. Haag, Lopuszanski and Sohnius extended the theorem by generalization of the notion of a Lie algebra to include algebraic systems whose defining relations involve in addition to the usual commutators also anticommutators [3]. These algebras are called superalgebras.

The simplest superalgebra involves only one set of the fermionic generators Q_α ([4],[5],[6])

$$
\begin{aligned}
[P^\mu, P^\nu] &= 0 && (1.1)\\
[M^{\mu\nu}, P^\lambda] &= i(\eta^{\nu\lambda}P^\mu - \eta^{\mu\lambda}P^\nu) && (1.2)\\
[M^{\mu\nu}, M^{\rho\sigma}] &= i(\eta^{\nu\rho}M^{\mu\sigma} + \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\mu\rho}M^{\nu\sigma} - \eta^{\nu\sigma}M^{\mu\rho}) && (1.3)\\
[P^\mu, Q_a] &= 0 = [P^\mu, Q_{\dot{a}}] && (1.4)\\
[M^{\mu\nu}, Q_a] &= -i(\sigma^{\mu\nu})_a{}^b Q_b && (1.5)\\
[M^{\mu\nu}, Q^{\dot{a}}] &= -i(\bar{\sigma}^{\mu\nu})^{\dot{a}}{}_{\dot{b}} Q^{\dot{b}} && (1.6)\\
\{Q_a, Q_b\} &= 0 = \{Q_{\dot{a}}, Q_{\dot{b}}\} && (1.7)\\
\{Q_a, Q_{\dot{b}}\} &= 2\sigma^\mu_{a\dot{b}} P_\mu && (1.8)
\end{aligned}
$$

In this set of equations we have omitted the generators of the internal symmetry group which commutes with P_μ, $M_{\mu\nu}$ but their presence is of course allowed by the two previously mentioned theorems. This symmetry group is the known $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.

An irreducible representation of the superalgebra has the number of fermionic states equal to the number of bosonic states. This can be proved with the help of equation (1.8) and realizing that the generators Q_a, $Q_{\dot{b}}$ change a bosonic state into a fermionic one and vice versa.

The supersymmetry predicts many new particles. Because the generators Q_a commute with the mass squared operator $P^2 = P_\mu P^\mu$ the particles and its superpartners poses equal masses. However, this does not agree with the observed phenomena therefore the supersymmetry must be broken.

The Minimal Supersymmetric Standard Model (MSSM) extends the Standard Model in a minimal manner. That means, incorporating only one set of SUSY generators into the theory. Moreover, in the MSSM one makes the minimal choice of the Higgs sector, works only with two Higgs doublets. The SUSY is broken explicitly by the so-called soft SUSY breaking mechanism which allows for terms causing no quadratic divergences.

Chapter 2

Superspace and Superfields

2.1 General superfield

It is possible to derive a supersymmetric theory without using the formalism of superspace. Such treatment is described in Martin's Supersymmetry primer [7]. But the more elegant way is to work in the superfield language. In this formalism the supersymmetry is inherently manifest like Lorentz invariance in four dimensional Minkowski space.

In supersymmetric models this four dimensional space is extended to superspace. Supercoordinates consist of the usual four Minkowski coordinates and of four anticommuting Grassman numbers that can be compactly written by the use of the two Weyl spinors.

$$\text{superspace coordinates: } (x^\mu, \theta_a, \theta_{\dot{a}}) \qquad a = 1,2 \qquad \dot{a} = \dot{1}, \dot{2}$$

A general superfield Φ is an operator-valued function defined on superspace and is understood in terms of its power series expansion in θ and $\bar{\theta}$. In addition, superfield is a Lorentz scalar or pseudoscalar as we want to build a supersymmetric lagrangian that is Lorentz invariant.

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) &= f(x) + \theta^a \phi_a(x) + \theta_{\dot{a}} \chi^{\dot{a}}(x) + (\theta\theta) m(x) + (\bar{\theta}\bar{\theta}) n(x) \\ &\quad + (\theta\sigma^\mu\bar{\theta}) V_\mu(x) + (\theta\theta)\theta_{\dot{a}} \lambda^{\dot{a}}(x) + (\bar{\theta}\bar{\theta})\theta^a \psi_a(x) \\ &\quad + (\theta\theta)(\bar{\theta}\bar{\theta}) d(x)\end{aligned} \tag{2.1}$$

The elements $f(x), \phi_a(x), \chi^{\dot{a}}(x) \ldots$ are called component fields.

An element of the subgroup of the supersymmetry group (when we omit $M_{\mu\nu}$) is

$$G(x,\theta,\bar{\theta}) = \exp[i(-x^\mu P_\mu + \theta Q + \bar{\theta}\bar{Q})] \tag{2.2}$$

Now we would like to construct linear representation of this group (of the superalgebra). We consider the right action induced in $(x^\mu, \theta_a, \theta_{\dot{a}})$ parameter space by the group elements

$$\begin{aligned}G(x,\theta,\bar{\theta})G(a,\xi,\bar{\xi}) &= \exp\Big[i\Big(-(x^\mu + a^\mu)P_\mu - i(\xi\sigma^\mu\bar{\theta})P_\mu + i(\theta\sigma^\mu\bar{\xi})P_\mu \\ &+ (\theta+\xi)Q + (\bar{\theta}+\bar{\xi})\bar{Q}\Big)\Big] \\ &= G(x^\mu + a^\mu + i(\xi\sigma^\mu\bar{\theta}) - i(\theta\sigma^\mu\bar{\xi}), \theta+\xi, \bar{\theta}+\bar{\xi}) \equiv G(B)\end{aligned} \tag{2.3}$$

where we have used the relation: $e^A e^B = e^{A+B+\frac{1}{2}[A,B]+\ldots}$

Thus the supersymmetric generators induce the motion in group parameter space: $(x, \theta, \bar{\theta}) \to (B)$. For a superfield that is a function on the superspace we have

$$\begin{aligned} \Phi(B) &= \Phi(x,\theta,\bar{\theta}) + (a^\mu + i\xi\sigma^\mu\bar{\theta} - i\theta\sigma^\mu\bar{\xi})\frac{\partial\Phi}{\partial x^\mu} + \xi^a\frac{\partial\Phi}{\partial\theta^a} + \xi_{\dot{a}}\frac{\partial\Phi}{\partial\theta_{\dot{a}}} + \ldots \\ &\stackrel{!}{=} \Big(1 - ia^\mu P_\mu + i\xi Q + i\bar{\xi}\bar{Q} + \ldots\Big)\Phi(x,\theta,\bar{\theta}) \end{aligned} \tag{2.4}$$

Hence the supersymmetric generators and momentum generator represented as differential operators are given by

$$\begin{aligned} P_\mu &= i\partial_\mu \\ iQ_a &= \frac{\partial}{\partial\theta^a} + i(\sigma^\mu)_{a\dot{b}}\theta^{\dot{b}}\partial_\mu \\ iQ^{\dot{a}} &= \frac{\partial}{\partial\theta_{\dot{a}}} + i(\bar{\sigma}^\mu)^{\dot{a}b}\theta_b\partial_\mu \end{aligned} \tag{2.5}$$

This is the linear representation of the supersymmetry algebra we were looking for. It can be explicitly verified that the generators satisfy the relations of superalgebra. Next we will show that the anticommuting relation (1.8) holds

$$\begin{aligned} \{Q_a, Q_{\dot{b}}\}\Phi &= -\{\partial_a + i\sigma^\mu{}_{a\dot{c}}\theta^{\dot{c}}\partial_\mu, -\partial_{\dot{b}} - i\theta^b\sigma^\nu{}_{b\dot{b}}\partial_\nu\}\Phi \\ &= \{\partial_a, \partial_{\dot{b}}\}\Phi + i\{\partial_a, \theta^b\sigma^\mu{}_{b\dot{b}}\partial_\mu\}\Phi + i\{\sigma^\mu{}_{a\dot{c}}\theta^{\dot{c}}\partial_\mu, \partial_{\dot{b}}\}\Phi \\ &- \{\sigma^\mu{}_{a\dot{a}}\theta^{\dot{c}}\partial_\mu, \theta^b\sigma^\nu{}_{b\dot{b}}\partial_\nu\}\Phi = 2\sigma^\mu{}_{a\dot{b}}P_\mu\Phi \end{aligned} \tag{2.6}$$

where for example

$$\begin{aligned} \{\partial^a, \theta^b\sigma^\mu{}_{b\dot{b}}\partial_\mu\}\Phi &= \partial_a[\theta^b\sigma^\mu{}_{b\dot{b}}\partial_\mu\Phi] + \theta^b\sigma^\mu{}_{b\dot{b}}\partial_\mu\partial_a\Phi \\ &= \delta_a^b\sigma^\mu{}_{b\dot{b}}\partial_\mu\Phi - \theta^b\sigma^\mu{}_{b\dot{b}}\partial_\mu\partial_a\Phi + \theta^b\sigma^\mu{}_{b\dot{b}}\partial_\mu\partial_a\Phi = i\sigma^\mu{}_{a\dot{b}}\partial_\mu\Phi \end{aligned} \tag{2.7}$$

The infinitesimal supersymmetric transformation (not considering momentum generator) can be seen from the relations (2.2) and (2.3)

$$\begin{aligned} \Phi &\to \Phi + \delta_S\Phi \\ \delta_S &= i(\xi Q + \bar{\xi}\bar{Q}) \end{aligned} \tag{2.8}$$

Next we define covariant derivatives

$$\begin{aligned} D_a &= \partial_a - i(\sigma^\mu)_{a\dot{b}}\theta^{\dot{b}}\partial_\mu \\ D^{\dot{a}} &= \partial^{\dot{a}} - i(\bar{\sigma}^\mu)^{\dot{a}b}\theta_b\partial_\mu \end{aligned} \tag{2.9}$$

The very important feature (hence its name) of covariant derivatives is that they commute with the infinitesimal supersymmetric transformation

$$\begin{aligned} [D_a, \delta_S] &= 0 \\ [D_{\dot{a}}, \delta_S] &= 0 \end{aligned} \tag{2.10}$$

The general superfield does not provide an irreducile representation of the supersymmetry algebra. By imposing the constraints that are covariant under the algebra we come to an irreducible representation. These constraints yield the following three types of superfields

$$\begin{array}{rclcl} D_{\dot a}\Phi(x,\theta,\bar\theta) &=& 0 & \rightarrow & \text{left-handed chiral (scalar) superfield} \\ D_a\Phi^\dagger(x,\theta,\bar\theta) &=& 0 & \rightarrow & \text{right-handed chiral (scalar) superfield} \\ \Phi(x,\theta,\bar\theta) &=& \Phi^\dagger(x,\theta,\bar\theta) & \rightarrow & \text{vector superfield} \end{array} \tag{2.11}$$

2.2 Chiral superfields

2.2.1 Left-handed chiral superfield

Our aim is to find out a superfield that satisfies the first condition in (2.11). It is easier to solve the constraint in terms of the new variables $(y,\theta',\bar\theta')$ where

$$\begin{aligned} y^\mu &= x^\mu - i\theta\sigma^\mu\bar\theta \\ \theta'_a &= \theta_a, \theta'_{\dot a} = \theta_{\dot a} \end{aligned} \tag{2.12}$$

The covariant derivative after transforming to this new variables is

$$D_{\dot a} = -\frac{\partial}{\partial\theta^{\dot a}}\bigg|_{y,\theta} \tag{2.13}$$

Now to get general solution it suffices (because $D_{\dot a}(x)y^\mu = 0, D_{\dot a}(x)\theta = 0$ and because the Φ cannot contain the variable $\bar\theta$ as the relation (2.13) indicates) to expand the superfield in the variables y,θ

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \tag{2.14}$$

where ϕ, F are complex scalar fields (giving four bosonic degrees of freedom) and ψ is left-handed Weyl spinor (four fermionic degrees of freedom). The dependence on the left-handed spinor is the origin for the name of left-handed superfield. We will see later that the field F plays the role of auxiliary field (does not have a dynamical term in lagrangian).

Transforming back to the original variables we obtain

$$\begin{aligned} \Phi(x,\theta,\bar\theta) &= \phi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) - i\partial_\mu\phi(x)(\theta\sigma^\mu\bar\theta) \\ &- i\sqrt{2}\theta\partial_\mu\psi(x)(\theta\sigma^\mu\bar\theta) - \frac{1}{2}\partial_\mu\partial_\nu\phi(x)(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) \end{aligned} \tag{2.15}$$

And using relations (A.8), (A.12) we come to the final expression

$$\begin{aligned} \Phi(x,\theta,\bar\theta) &= \phi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) - i\partial_\mu\phi(x)(\theta\sigma^\mu\bar\theta) \\ &+ \frac{i}{\sqrt{2}}(\theta\theta)\partial_\mu\psi(x)\sigma^\mu\bar\theta - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial_\mu\partial^\mu\phi(x) \end{aligned} \tag{2.16}$$

2.2.2 Right-handed chiral superfield

Right-handed superfield satisfies the second condition in (2.11). One can derive the solution in analogous way to previous subsection. The new variables are

$$\begin{aligned} z^\mu &= x^\mu + i\theta\sigma^\mu\bar{\theta} \\ \theta'_a &= \theta_a, \theta'_{\dot{a}} = \theta_{\dot{a}} \end{aligned} \tag{2.17}$$

Then the covariant derivative transforms to be

$$D_a = \left.\frac{\partial}{\partial\theta^a}\right|_{z,\bar{\theta}} \tag{2.18}$$

Expansion of right-handed superfield in new variables is

$$\Phi^\dagger(z,\bar{\theta}) = \phi^*(z) + \sqrt{2}\bar{\theta}\bar{\psi}(z) + \bar{\theta}\bar{\theta}F^*(z) \tag{2.19}$$

And after transforming back to the original variables the final expression is as follows

$$\begin{aligned} \Phi^\dagger(x,\theta,\bar{\theta}) &= \phi^*(x) + \sqrt{2}\bar{\theta}\bar{\psi}(x) + \bar{\theta}\bar{\theta}F^*(x) + i\partial_\mu\phi^*(x)\theta\sigma^\mu\bar{\theta} \\ &- \frac{i}{\sqrt{2}}(\bar{\theta}\bar{\theta})\theta\sigma^\mu\partial_\mu\bar{\psi}(x) - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial_\mu\partial^\mu\phi^*(x) \end{aligned} \tag{2.20}$$

We finally remark that z^μ is complex conjugate of y^μ and (after realizing the identity (A.7)) the superfield $\Phi^\dagger$ is conjugate of the superfield Φ as one could anticipate earlier.

Next we show sum of left and right chiral superfield which will become important later by definition of gauge transformation of vector superfield.

$$\begin{aligned} \Phi(x) + \Phi^\dagger(x) &= \phi(x) + \phi^*(x) + \sqrt{2}\theta\psi(x) + \sqrt{2}\bar{\theta}\bar{\psi}(x) \\ &+ (\theta\theta)F(x) + (\bar{\theta}\bar{\theta})F^*(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\Big(\phi^*(x) - \phi(x)\Big) \\ &- \frac{i}{\sqrt{2}}(\theta\theta)\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x) - \frac{i}{\sqrt{2}}(\bar{\theta}\bar{\theta})\theta\sigma^\mu\partial_\mu\bar{\psi}(x) \\ &- \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial_\mu\partial^\mu\Big(\phi(x) + \phi^*(x)\Big) \end{aligned} \tag{2.21}$$

2.2.3 Supersymmetric transformation of component fields

The transformation law for a general superfield is defined by components. In this subsection we will demonstrate as an example the transformation of the components of the left-handed chiral superfield

$$\delta_S\Phi(y) = \delta_S\phi(y) + \sqrt{2}\theta\delta_S\psi(y) + \theta\theta\delta_S F(y) \tag{2.22}$$

It is easier to work with the generators Q (2.5) written in variable y

$$\begin{aligned} iQ_a(y) &= \partial_a \\ iQ^{\dot{a}}(y) &= \partial^{\dot{a}} - 2i\theta^b\sigma^\mu{}_b{}^{\dot{a}}\partial_\mu \end{aligned} \tag{2.23}$$

When we apply the last two equations on the relation (2.8) we obtain

$$\delta_S\Phi = \sqrt{2}\xi\psi + 2\xi\theta F + i\sqrt{2}(\theta\theta)(\partial_\mu\psi\sigma^\mu\bar{\xi}) - 2i(\theta\sigma^\mu\bar{\xi})\partial_\mu\phi \tag{2.24}$$

And after the comparison of the terms standing by the equal power of Grassman number θ in relations (2.22), (2.24) we finally come to the transformed component fields

$$\begin{aligned}
\delta_S\phi &= \sqrt{2}\xi\psi && (2.25)\\
\delta_S\psi_a &= \sqrt{2}\xi_a F - i\sqrt{2}\sigma^\mu{}_{a\dot{b}}\xi^{\dot{b}}\partial_\mu\phi && (2.26)\\
\delta_S F &= i\sqrt{2}(\partial_\mu\psi\sigma^\mu\bar{\xi}) && (2.27)
\end{aligned}$$

We observe that the field F transforms into a total divergence. This will become important when constructing supersymmetric lagrangian implying its invariance under supersymmetric transformation.

2.2.4 Products of chiral superfields

For the construction of supersymmetric lagrangian one needs the products $\Phi_i\Phi_j$, $\Phi_i^\dagger\Phi_j$, $\Phi_i\Phi_j\Phi_k$ where indices i, j, k distinguish various superfields. The higher products lead to nonrenormalizable theories therefore we will not consider them.

The product of chiral superfields of the equal handedness is a chiral superfield of the same handedness

$$\begin{aligned}
\Phi_i(y,\theta)\Phi_j(y,\theta) &= \phi_i(y)\phi_j(y) + \sqrt{2}\theta\Big(\psi_i(y)\phi_j(y) + \phi_i(y)\psi_j(y)\Big)\\
&+ \theta\theta\Big(\phi_i(y)F_j(y) + \phi_j(y)F_i(y) - \psi_i(y)\psi_j(y)\Big) \qquad (2.28)
\end{aligned}$$

$$\begin{aligned}
&\Phi_i(y,\theta)\Phi_j(y,\theta)\Phi_k(y,\theta) = \phi_i(y)\phi_j(y)\phi_k(y)\\
&\quad+\sqrt{2}\theta\Big(\psi_i(y)\phi_j(y)\phi_k(y) + \phi_i(y)\psi_j(y)\phi_k(y) + \phi_i(y)\phi_j(y)\psi_k(y)\Big)\\
&\quad+\theta\theta\Big(\phi_i(y)\phi_j(y)F_k(y) + \phi_i(y)F_j(y)\phi_k(y) + F_i(y)\phi_j(y)\phi_k(y)\\
&\quad-\psi_i(y)\psi_j(y)\phi_k(y) - \psi_i(y)\psi_k(y)\phi_j(y) - \psi_j(y)\psi_k(y)\phi_i(y)\Big) \qquad (2.29)
\end{aligned}$$

$$\begin{aligned}
\Phi_i^\dagger(x)\Phi_j(x) &= \ldots - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Big[\phi_i^*(x)\partial_\mu\partial^\mu\phi_j(x) + \phi_j(x)\partial_\mu\partial^\mu\phi_i^*(x)\Big]\\
&+ (\theta\sigma^\mu\bar{\theta})(\theta\sigma^\nu\bar{\theta})(\partial_\mu\phi_i^*(x))(\partial_\nu\phi_j(x)) - i(\bar{\theta}\bar{\theta})(\theta\sigma^\mu\partial_\mu\bar{\psi}_i(x))(\theta\psi_j(x))\\
&+ i(\bar{\theta}\bar{\psi}_i(x))(\theta\theta)(\partial_\mu\psi_j(x)\sigma^\mu\bar{\theta}) + (\theta\theta)(\bar{\theta}\bar{\theta})F_i^*(x)F_j(x)\\
&= \ldots + (\theta\theta)(\bar{\theta}\bar{\theta})\Big[F_i^*(x)F_j(x) - \phi_i^*(x)(\partial_\mu\partial^\mu\phi_j(x))\\
&- i(\partial_\mu\bar{\psi}_i(x)\bar{\sigma}^\mu\psi_j(x))\Big] \qquad (2.30)
\end{aligned}$$

The last product is not a chiral superfield but vector superfield treated in the following section. We have explicitly showed only the $(\theta\theta)(\bar{\theta}\bar{\theta})$ - component as this component in general superfield also transforms into a total divergence under a supersymmetric transformation and therefore becomes important for the construction of the supersymmetric invariant lagrangian.

2.3 Vector superfield

The third type of a superfield is a vector superfield. In deriving its component expansion we start from the expression for a general superfield

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) &= C(x)+\theta^a\phi_a(x)+\theta_{\dot{a}}\chi^{\dot{a}}(x)+(\theta\theta)M(x)+(\bar{\theta}\bar{\theta})N(x)\\ &\quad +(\theta\sigma^\mu\bar{\theta})V_\mu(x)+(\theta\theta)\theta_{\dot{a}}\lambda^{\dot{a}}(x)+(\bar{\theta}\bar{\theta})\theta^a\psi_a(x)\\ &\quad +(\theta\theta)(\bar{\theta}\bar{\theta})D(x)\end{aligned} \tag{2.31}$$

If we require $\Phi(x,\theta,\bar{\theta})=\Phi^\dagger(x,\theta,\bar{\theta})$ then we are left with the following restrictions

$$C=C^* \quad V_\mu=V_\mu^* \quad D=D^* \tag{2.32}$$

$$M^*=N \quad \phi=\chi \quad \lambda=\psi \tag{2.33}$$

The vector superfield thus consists of two Weyl spinors λ,χ (eight real fermionic degrees of freedom), two real scalars C,D, one complex scalar M and one real vector V_μ (giving altogether eight real bosonic degrees of freedom).

Now we rewrite the vector superfield in a more convenient way in which certain components of V are invariant under gauge transformation that will be defined below. We achieve this by sending

$$\lambda(x) \to \lambda(x)-\frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x) \tag{2.34}$$

$$D(x) \to \frac{1}{2}D(x)-\frac{1}{4}\partial_\mu\partial^\mu C(x) \tag{2.35}$$

The helpful relation one will need is $(\sigma^\mu{}_{a\dot{b}})^\dagger=\sigma^\mu{}_{b\dot{a}}$. Finally we obtain

$$\begin{aligned}V(x,\theta,\bar{\theta}) &= C(x)+\theta\chi(x)+\bar{\theta}\bar{\chi}(x)+(\theta\theta)M(x)+(\bar{\theta}\bar{\theta})M^*(x)\\ &+ \theta\sigma^\mu\bar{\theta}V_\mu(x)+(\theta\theta)\bar{\theta}\Big(\bar{\lambda}(x)-\frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\Big)\\ &+ (\bar{\theta}\bar{\theta})\theta\Big(\lambda(x)-\frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\Big)+(\theta\theta)(\bar{\theta}\bar{\theta})\left(\frac{1}{2}D(x)-\frac{1}{4}\partial_\mu\partial^\mu C(x)\right)\end{aligned} \tag{2.36}$$

The supersymmetric generalization of a gauge transformation is defined as

$$\begin{aligned}V(x,\theta,\bar{\theta}) &\to V(x,\theta,\bar{\theta})+\Phi(x,\theta,\bar{\theta})+\Phi^\dagger(x,\theta,\bar{\theta})\\ &\to V(x,\theta,\bar{\theta})+i\Lambda(x,\theta,\bar{\theta})-i\Lambda^\dagger(x,\theta,\bar{\theta})\end{aligned} \tag{2.37}$$

where Φ is some left-handed chiral superfield. In component fields the gauge transformation gives

$$\begin{aligned} C\to C+\phi+\phi^* &\qquad V_\mu\to V_\mu+i\partial_\mu(\phi^*-\phi)\\ \chi\to\chi+\sqrt{2}\psi &\qquad \lambda\to\lambda\\ M\to M+F &\qquad D\to D\end{aligned} \tag{2.38}$$

We see that the component fields λ,D are invariant under the gauge transformation. Equally important observation is that vector component V_μ transforms into a gradient.

This motivated us to define the gauge transformation in the presented form.

From the set of equations (2.38) we see that we can choose particular scalar field Φ (that is particular gauge) in which the fields C, χ, M vanish. This gauge is called Wess-Zumino gauge. We remark that this gauge does not fix the imaginary part of the field ϕ, we are free to set it zero.

When we finally redefine $\lambda \rightarrow -i\lambda$ and $\bar{\lambda} \rightarrow i\bar{\lambda}$ we get

$$V_{WZ}(x,\theta,\bar{\theta}) = \theta\sigma^{\mu}\bar{\theta}V_{\mu}(x) + i(\theta\theta)\bar{\theta}\bar{\lambda}(x) - i(\bar{\theta}\bar{\theta})\theta\lambda(x) + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})D(x) \tag{2.39}$$

The advantage of the Wess-Zumino gauge is that the third and higher powers of the vector superfield V equal zero. Then for instance

$$\begin{aligned} \exp(V) &= 1 + V + \frac{1}{2}V^2 = 1 + \theta\sigma^{\mu}\bar{\theta}V_{\mu}(x) + i(\theta\theta)\bar{\theta}\bar{\lambda}(x) - i(\bar{\theta}\bar{\theta})\theta\lambda(x) \\ &+ (\theta\theta)(\bar{\theta}\bar{\theta})\Big(\frac{1}{2}D(x) + \frac{1}{4}V^{\mu}(x)V_{\mu}(x)\Big) \end{aligned} \tag{2.40}$$

2.4 Field strength superfield

The supersymmetric field strength for an arbitrary vector superfield V is defined by its components

$$\begin{aligned} W_a &= -\frac{1}{4}(\bar{D}\bar{D})D_a V \\ W_{\dot{a}} &= -\frac{1}{4}(DD)D_{\dot{a}} V \end{aligned} \tag{2.41}$$

These spinor superfields are chiral ones (they satisfy the first two constraints in (2.11)). With the help of the relation $\{\bar{D}, D\}\Phi \sim P_{\mu}\Phi$ and $[\bar{D}, P_{\mu}] = 0$ one can prove that the superfields are also gauge invariant.

When expanding the superfield W_a the most plausible way is to calculate it in the variable y since the two covariant derivatives $(\bar{D}\bar{D})$ become very simple. We start our calculating with

$$\begin{aligned} D_a V(y) &= (\partial_a - 2i\sigma^{\mu}{}_{a\dot{b}}\theta^{\dot{b}}\partial_{\mu})\Big[\theta\sigma^{\nu}\bar{\theta}V_{\nu}(y) + i(\theta\theta)\bar{\theta}\bar{\lambda}(y) - i(\bar{\theta}\bar{\theta})\theta\lambda(y) \\ &+ \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})\Big(D(y) + i\partial_{\mu}V^{\mu}(y)\Big)\Big] \end{aligned} \tag{2.42}$$

with the realization of the following relation

$$-\frac{1}{4}\bar{D}\bar{D}(\bar{\theta}\bar{\theta}) = -\frac{1}{4}(-\partial_{\dot{a}}\partial^{\dot{a}})(\bar{\theta}\bar{\theta}) = \frac{1}{2}\partial_{\dot{a}}\theta^{\dot{a}} = 1 \tag{2.43}$$

it is not complicated to come to the expanded form of the field strength W_a

$$\begin{aligned} W_a(y) &= -i\lambda_a(y) + \theta_a D(y) - (\theta\theta)\sigma^{\mu}{}_{a\dot{c}}\partial_{\mu}\bar{\lambda}^{\dot{c}}(y) \\ &+ i\sigma^{\mu}{}_{a\dot{b}}\varepsilon^{\dot{b}\dot{c}}\theta^{b}\sigma^{\nu}{}_{b\dot{c}}\partial_{\mu}V_{\nu}(y) + i\theta_a\partial_{\mu}V^{\mu}(y) \end{aligned} \tag{2.44}$$

This is still not the final form because the third term can be rewritten and its one part will cancel the fourth term

$$
\begin{aligned}
i\sigma^{\mu}{}_{a\dot{b}}\varepsilon^{\dot{b}\dot{c}}\theta^{b}\sigma^{\nu}{}_{b\dot{c}}\partial_{\mu}V_{\nu}(y) &= i\varepsilon^{bd}\theta_{d}\varepsilon^{\dot{b}\dot{c}}\sigma^{\mu}{}_{b\dot{c}}\sigma^{\nu}{}_{a\dot{b}}\partial_{\mu}V_{\nu}(y) = -i\theta_{d}\sigma^{\mu}{}_{a\dot{b}}\bar{\sigma}^{\nu\,\dot{b}d}\partial_{\mu}V_{\nu}(y) \\
&= -i\theta_{a}\partial_{\mu}V^{\mu}(y) - 2\sigma^{\mu\nu}{}_{a}{}^{d}\theta_{d}\partial_{\mu}V_{\nu}(y) \qquad (2.45)
\end{aligned}
$$

where we have used the relation (A.10). In an analogous way one can calculate the expansion of the superfield $W_{\dot{a}}$

$$
\begin{aligned}
W_{a}(y) &= -i\lambda_{a}(y) + \theta_{a}D(y) - (\sigma^{\mu\nu}\theta)_{a}V_{\mu\nu}(y) - (\theta\theta)(\sigma^{\mu}\partial_{\mu}\bar{\lambda}(y))_{a} \\
W_{\dot{a}}(z) &= +i\lambda_{\dot{a}}(z) + \theta_{\dot{a}}D(z) + \varepsilon_{\dot{a}\dot{b}}(\bar{\sigma}^{\mu\nu}\bar{\theta})^{\dot{b}}V_{\mu\nu}(z) - (\bar{\theta}\bar{\theta})(\partial_{\mu}\lambda(z)\sigma^{\mu})_{\dot{a}} \qquad (2.46)
\end{aligned}
$$

where $V_{\mu\nu} = \partial_{\mu}V_{\nu} - \partial_{\nu}V_{\mu}$ is the field strength tensor we are used to from the Standard Model.

2.5 Supersymmetric lagrangian - the abelian case

The most general and renormalizable lagrangian that includes chiral as well as vector superfields is

$$
\mathscr{L} = \mathscr{L}_{\Phi} + \mathscr{L}_{W} \qquad (2.47)
$$

where

$$
\begin{aligned}
\mathscr{L}_{\Phi} &= \Phi_{i}^{\dagger}\Phi_{i}\Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \Big[\Big(\frac{1}{2}m_{ij}\Phi_{i}\Phi_{j} + \frac{1}{3}b_{ijk}\Phi_{i}\Phi_{j}\Phi_{k} + c_{i}\Phi_{i}\Big)\Big|_{\theta\theta} + \text{h.c.}\Big] \qquad (2.48) \\
\mathscr{L}_{W} &= \frac{1}{4}\Big(W^{a}W_{a}|_{\theta\theta} + W_{\dot{a}}W^{\dot{a}}|_{\bar{\theta}\bar{\theta}}\Big) \qquad (2.49)
\end{aligned}
$$

The constants m_{ij}, b_{ijk} are symmetric in their indices. Since W_a is chiral, W^aW_a is a scalar field and therefore the $\theta\theta$-component is of interest in our construction as this component transforms into a derivative. Expansion of the supersymmetric invariant lagrangian density $\mathscr{L}_{\Phi}$ to components yields

$$
\begin{aligned}
\mathscr{L}_{\Phi} &= i\bar{\psi}_{i}(x)\bar{\sigma}^{\mu}\partial_{\mu}\psi_{i}(x) - \phi_{i}^{*}(x)\partial_{\mu}\partial^{\mu}\phi_{i}(x) + F_{i}^{*}(x)F_{i}(x) \\
&+ \Big[m_{ik}\phi_{k}(x)F_{k}(x) + b_{ijk}\phi_{i}(x)\phi_{j}(x)F_{k}(x) \\
&- \frac{1}{2}m_{ij}\psi_{i}(x)\psi_{j}(x) - b_{ijk}\psi_{i}(x)\psi_{j}(x)\phi_{k}(x) + c_{i}F_{i}(x) + \text{h.c.}\Big] \qquad (2.50)
\end{aligned}
$$

When deriving the lagrangian density $\mathscr{L}_{W}$ one needs to use identities (A.1), (A.5) and (A.9). With their help we obtain

$$
\begin{aligned}
W^{a}W_{a}\Big|_{\theta\theta} &= D^{2}(x) + 2i\lambda(x)\sigma^{\mu}\partial_{\mu}\bar{\lambda}(x) - \frac{1}{2}V_{\mu\nu}(x)V^{\mu\nu}(x) - \frac{i}{4}\varepsilon^{\mu\nu\rho\sigma}V_{\mu\nu}V_{\rho\sigma} \qquad (2.51) \\
W_{\dot{a}}W^{\dot{a}}\Big|_{\bar{\theta}\bar{\theta}} &= D^{2}(x) - 2i\partial_{\mu}\lambda(x)\sigma^{\mu}\bar{\lambda}(x) - \frac{1}{2}V_{\mu\nu}(x)V^{\mu\nu}(x) + \frac{i}{4}\varepsilon^{\mu\nu\rho\sigma}V_{\mu\nu}V_{\rho\sigma} \qquad (2.52)
\end{aligned}
$$

Then the supersymmetric and gauge invariant hermitian lagrangian density $\mathscr{L}_{W}$ is given by

$$
\mathscr{L}_{W} = i\lambda(x)\sigma^{\mu}\partial_{\mu}\bar{\lambda}(x) + \frac{1}{2}D^{2}(x) - \frac{1}{4}V_{\mu\nu}(x)V^{\mu\nu}(x) \qquad (2.53)
$$

Here we can summarize what our lagrangian $\mathscr{L}$ describes. In $\mathscr{L}_\Phi$ we have got a kinetic term for the fermion ψ as well as for the sfermion ϕ. We have also got an interaction between this two fields. $\mathscr{L}_W$ contains kinetic term for the gaugino λ as well as for the vector particle V. Here is the proper time to remark that the kinetic term for ψ (λ) is a part of the Dirac equation (Majorana equation) with the sign as in the book of Peskin, Schroeder [8]. This was one of our goals and we also remark that this sign was the reason why we have chosen the action from the right at the very beginning (see (2.3)).

The fields F, D are auxiliary fields and can be replaced by considering the Euler-Lagrange equation which further leads to new interaction terms.

The whole lagrangian does not describe the interaction of matter particles (ϕ, ψ) with gauge particles (V, λ). This interaction will occur when we consider the non-abelian case.

Chapter 3

Supersymmetric non-abelian gauge theories

3.1 The most general lagrangian

In this chapter we will discuss the gauge invariant interactions of chiral and vector superfields.
Let G be a compact gauge group with Lie algebra $\mathcal{G}$. Under a local gauge transformation the left and right handed chiral superfields change to

$$\Phi_i' = e^{-i2g_i\Lambda(x)}\Phi_i \qquad\qquad (\Phi_i')^\dagger = \Phi_i^\dagger e^{i2g_i\Lambda^\dagger(x)} \tag{3.1}$$

where Λ is a matrix

$$\Lambda_{ij} = T_{ij}^{(a)}\Lambda^{(a)} \tag{3.2}$$

and the superfields $\Lambda^{(a)}$ satisfy the chirality condition (2.11) so that transformed Φ remains chiral.
The matrices $T^{(a)}$ are the hermitian generators of the gauge group in the particular representation that is defined by the chiral field Φ. In the adjoint representation we normalize the generators as follows

$$\mathrm{Tr}(T^{(a)}T^{(b)}) = k\delta^{ab}, \qquad k > 0 \tag{3.3}$$

and the commutator of the generators is

$$[T^{(a)}, T^{(b)}] = it^{abc}T^{(c)} \tag{3.4}$$

where t^{abc} are the structure constants of the gauge group G.

The kinetic term $\Phi^\dagger\Phi$ in the lagrangian (2.48) is not invariant under the local gauge transformation. We are forced to introduce the vector superfield V provided also that we extent the transformation law (2.37)

$$e^{2g_iV'(x)} = e^{-i2g_i\Lambda^\dagger(x)}e^{2g_iV(x)}e^{i2g_i\Lambda(x)} \tag{3.5}$$

where the superfield V is a matrix as well

$$V_{ij} = V^{(a)} T^{(a)}_{ij} \tag{3.6}$$

With this addition, the lagrangian $\mathscr{L}'_\Phi$ is

$$\mathscr{L}'_\Phi = \Phi_i^\dagger e^{2g_i V}\Phi_i\Big|_{\theta\theta\bar\theta\bar\theta} + \left[\left(\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}b_{ijk}\Phi_i\Phi_j\Phi_k + c_i\Phi_i\right)\Big|_{\theta\theta} + \text{h.c.}\right] \tag{3.7}$$

To obtain gauge invariant theory we also have to satisfy the demands of the following relations

$$\begin{aligned} c_i &= 0 \quad \text{if} \quad g_i \neq 0 \\ m_{ij} &= 0 \quad \text{if} \quad g_i + g_j \neq 0 \\ b_{ijk} &= 0 \quad \text{if} \quad g_i + g_j + g_k \neq 0 \end{aligned} \tag{3.8}$$

We are also forced to generalize the prescription for the field strength superfields

$$W_a = -\frac{1}{4}\bar D\bar D e^{-2gV} D_a e^{2gV} \tag{3.9}$$

$$W_{\dot a} = -\frac{1}{4}DD\Big(D_{\dot a}e^{2gV}\Big)e^{-2gV} \tag{3.10}$$

Under the gauge transformation the $W_a, W_{\dot a}$ transform in the following way

$$W'_a = e^{-i2g\Lambda} W_a e^{i2g\Lambda} \tag{3.11}$$

$$W'_{\dot a} = e^{-i2g\Lambda^\dagger} W_{\dot a} e^{i2g\Lambda^\dagger} \tag{3.12}$$

Here we explicitly show the gauge transformation of $W_{\dot a}$

$$\begin{aligned} W'_{\dot a} &\sim DD[D_{\dot a}(e^{-i2g\Lambda^\dagger}e^{2gV}e^{i2g\Lambda})]e^{-i2g\Lambda}e^{-2gV}e^{i2g\Lambda^\dagger} = \\ &= DD[D_{\dot a}(e^{-i2g\Lambda^\dagger}e^{2gV})]e^{-2gV}e^{i2g\Lambda^\dagger} = \\ &= DD(D_{\dot a}e^{-i2g\Lambda^\dagger})e^{i2g\Lambda^\dagger} + DDe^{-i2g\Lambda^\dagger}(D_{\dot a}e^{2gV})e^{-2gV}e^{i2g\Lambda^\dagger} \end{aligned} \tag{3.13}$$

The second term gives the required result while the first term equals zero

$$\begin{aligned} DD(D_{\dot a}e^{-i2g\Lambda^\dagger})e^{i2g\Lambda^\dagger} &= DD(D_{\dot a}1) - DD[e^{-i2g\Lambda^\dagger}D_{\dot a}e^{i2g\Lambda^\dagger}] \\ &= -e^{-i2g\Lambda^\dagger}D^a\{D_a, D_{\dot a}\}e^{i2g\Lambda^\dagger} \\ &= -e^{-i2g\Lambda^\dagger}2\sigma^\mu{}_{a\dot a}P_\mu D^a e^{i2g\Lambda^\dagger} = 0 \end{aligned} \tag{3.14}$$

Now we are ready to write down the full supersymmetric and local gauge invariant lagragian that describes renormalizable interaction of scalar, spinor and vector fields

$$\mathscr{L} = \frac{1}{16kg^2}\text{Tr}\Big(W^a W_a\Big|_{\theta\theta} + W_{\dot a}W^{\dot a}\Big|_{\bar\theta\bar\theta}\Big) + \Phi^\dagger e^{2gV}\Phi\Big|_{\theta\theta\bar\theta\bar\theta} + \Big(W + \text{h.c.}\Big) \tag{3.15}$$

where W is called the superpotential and represents the products of chiral superfields.

3.2 Component expansion of the lagrangian

Our task now is to expand the lagrangian (3.15) into the components. At the end of the long journey we would like to recognize for instance the Dirac equation for the Dirac bispinor which will appear from the combination of the two Weyl spinors ψ as well as the Majorana equation for gaugino λ, then we will obtain kinetic term for the sfermion ϕ and for vector particle V^{μ} and finally, we will get the interaction of the vector, fermion and sfermion fields.

3.2.1 The term containing superfields $\Phi, \Phi^{\dagger}$

Here we write again the expansion of the left and right handed superfield as well as the expansion of $e^{2gV(x)}$

$$\begin{aligned}
\Phi(x^{\mu},\theta,\bar{\theta}) &= \phi(x)+\sqrt{2}\theta\psi(x)+\theta\theta F(x)-i\partial_{\mu}\phi(x)\theta\sigma^{\mu}\bar{\theta}\\
&+ \frac{i}{\sqrt{2}}(\theta\theta)\partial_{\mu}\psi(x)\sigma^{\mu}\bar{\theta}-\frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial_{\mu}\partial^{\mu}\phi(x) \qquad (3.16)\\
\Phi^{\dagger}(x^{\mu},\theta,\bar{\theta}) &= \phi^{*}(x)+\sqrt{2}\bar{\theta}\bar{\psi}(x)+\bar{\theta}\bar{\theta}F^{*}(x)+i\partial_{\mu}\phi^{*}(x)\theta\sigma^{\mu}\bar{\theta}\\
&- \frac{i}{\sqrt{2}}(\bar{\theta}\bar{\theta})\theta\sigma^{\mu}\partial_{\mu}\bar{\psi}(x)-\frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial_{\mu}\partial^{\mu}\phi^{*}(x) \qquad (3.17)\\
e^{2gV(x)} &= 1+2gV+2g^{2}V^{2}=1+2g(\theta\sigma^{\mu}\bar{\theta})V_{\mu}(x)+i2g(\theta\theta)\bar{\theta}\bar{\lambda}(x)\\
&- i2g(\bar{\theta}\bar{\theta})\theta\lambda(x)+(\theta\theta)(\bar{\theta}\bar{\theta})\Big(gD(x)+g^{2}V^{\mu}(x)V_{\mu}(x)\Big) \qquad (3.18)
\end{aligned}$$

Now we need to make a product of these three long expressions. Part of this product has been already counted when we have dealt with the $\Phi\Phi^{\dagger}$ (see first three terms in (2.50)). We also mention that now we cannot forget that V is a matrix and $\Phi_i^{\dagger}, \Phi_j$, ($i,j=1$ for U(1), $i,j=1,2$ for SU(2), $i,j=1,2,3$ for SU(3)) is a line, respectively a column. The remaining terms are

$$\begin{aligned}
\sqrt{2}(\bar{\theta}\bar{\psi}_i)2g(\theta\sigma^{\mu}\bar{\theta})V_{\mu}^{(a)}T_{ij}^{(a)}\sqrt{2}(\theta\psi_j)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} &= -gT_{ij}^{(a)}V_{\mu}^{(a)}(\bar{\psi}_i\bar{\sigma}^{\mu}\psi_j) \qquad (3.19)\\
i\partial_{\mu}\phi_i^{*}(\theta\sigma^{\mu}\bar{\theta})2g(\theta\sigma^{\nu}\bar{\theta})V_{\nu}^{(a)}T_{ij}^{(a)}\phi_j\Big|_{\theta\theta\bar{\theta}\bar{\theta}} &= igT_{ij}^{(a)}V_{\mu}^{(a)}(\partial^{\mu}\phi_i^{*})\phi_j \qquad (3.20)\\
\phi_i^{*}2g(\theta\sigma^{\mu}\bar{\theta})V_{\mu}^{(a)}T_{ij}^{(a)}(-i)\partial_{\nu}\phi_j(\theta\bar{\sigma}^{\nu}\bar{\theta})\Big|_{\theta\theta\bar{\theta}\bar{\theta}} &= -igT_{ij}^{(a)}V_{\mu}^{(a)}\phi_i^{*}(\partial^{\mu}\phi_j) \qquad (3.21)\\
\sqrt{2}(\bar{\theta}\bar{\psi}_i)i2g(\theta\theta)(\bar{\theta}\bar{\lambda}^{(a)})T_{ij}^{(a)}\phi_j\Big|_{\theta\theta\bar{\theta}\bar{\theta}} &= -i\sqrt{2}gT_{ij}^{(a)}(\bar{\lambda}^{(a)}\bar{\psi}_i)\phi_j \qquad (3.22)\\
\phi_i^{*}(-i2g)(\bar{\theta}\bar{\theta})(\theta\lambda^{(a)})T_{ij}^{(a)}\sqrt{2}(\theta\psi_j)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} &= i\sqrt{2}gT_{ij}^{(a)}\phi_i^{*}(\lambda^{(a)}\psi_j) \qquad (3.23)\\
g^{2}(T^{(a)}T^{(b)})_{ij}V_{\mu}^{(a)}V^{\mu(b)}\phi_i^{*}\phi_j &+ gT_{ij}^{(a)}D^{(a)}\phi_i^{*}\phi_j \qquad (3.24)
\end{aligned}$$

The resulting terms agrees with the terms published in the work of Haber and Kane [9] in appendix B, equation (B2).

3.2.2 The term containing field strength suprefields $W_a, W_{\dot{a}}$

The superfield $W_a(y)$ consists of the terms appearing in relation (2.46) multiplied by the factor $2g$ and of the addtional terms that arise because of the additional terms in the definition (3.9). So now the superfield W_a that is now a matrix consists of the following components

$$\begin{aligned} W_a(y) &= -i2g\lambda_a(y) + 2g\theta_a D(y) - 2g(\sigma^{\mu\nu}\theta)_a V_{\mu\nu}(y) - 2g(\theta\theta)(\sigma^\mu\partial_\mu\bar{\lambda}(y))_a \\ &+ g\theta_a V_\mu(y)^{(a)} V^{\mu\ (b)}(y) T^{(a)} T^{(b)} - i2g\sigma^{\mu\nu}{}_a{}^d\theta_d V_\mu^{(a)}(y) V_\nu^{(b)}(y) T^{(a)} T^{(b)} \\ &+ ig(\theta\theta)\sigma^\mu{}_{a\dot{b}}\bar{\lambda}^{\dot{b}\ (a)}(y) V_\mu^{(b)}(y)[T^{(a)}, T^{(b)}] \end{aligned} \quad (3.25)$$

Yet we need to calculate the Lorentz scalar term $W^a W_a$. The semi-result is

$$\begin{aligned} W^a W_a(x)\Big|_{\theta\theta} &= i4g^2(\lambda^{(a)}\sigma^\mu\bar{\partial}_\mu\lambda^{(b)})(T^{ab} + T^{ba}) \\ &+ 4g^3(\lambda^{(a)}\sigma^\mu\bar{\lambda}^{(b)})V_\mu^{(c)}\left(T^{a[b,c]} + T^{[b,c]a}\right) \\ &- g^2(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho} + i\varepsilon^{\mu\nu\rho\sigma})V_{\mu\nu}^{(a)} V_{\rho\sigma}^{(b)} T^{ab} \\ &+ i2g^3(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho} + i\varepsilon^{\mu\nu\rho\sigma})V_{\mu\nu}^{(a)} V_\sigma^{(b)} V_\rho^{(c)}(T^{abc} + T^{bca}) \\ &- 4g^4(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho} + i\varepsilon^{\mu\nu\rho\sigma})V_\mu^{(a)} V_\nu^{(b)} V_\sigma^{(c)} V_\rho^{(d)} T^{abcd} \\ &+ 4g^2 D^{(a)} D^{(b)} T^{ab} \end{aligned} \quad (3.26)$$

where $T^{ab} = T^{(a)}T^{(b)}, T^{[b,c]} = T^{(b)}T^{(c)} - T^{(c)}T^{(b)}$, etc.

From the $W_{\dot{a}}W^{\dot{a}}\Big|_{\bar{\theta}\bar{\theta}}$ we would obtain a similar result. The only difference would be the opposite signs standing by the terms containing $i\varepsilon^{\mu\nu\rho\sigma}$. Therefore these terms will not be present in our final lagrangian.
As a last step we would like to make a trace. Before doing so we have to slightly modify the last expression with the help of following equations

$$\begin{aligned} (\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho})V_{\mu\nu}^{(a)} V_\sigma^{(b)} V_\rho^{(c)} T^{abc} &= (V_{\mu\nu}^{(a)} V^{\nu(b)} V^{\mu(c)} - V_{\mu\nu}^{(a)} V^{\mu(b)} V^{\nu(c)}) T^{abc} \\ &= (V_{\mu\nu}^{(a)} V^{\nu(b)} V^{\mu(c)}) T^{a[b,c]} \end{aligned} \quad (3.27)$$

$$\begin{aligned} &(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho})V_\mu^{(a)} V_\nu^{(b)} V_\sigma^{(c)} V_\rho^{(d)} T^{abcd} = \\ &= (V_\mu^{(a)} V_\nu^{(b)} V^{\nu(c)} V^{\mu(d)} - V_\mu^{(a)} V_\nu^{(b)} V^{\mu(c)} V^{\nu(d)}) T^{abcd} = -V_\mu^{(a)} V_\nu^{(b)} V^{\mu(c)} V^{\nu(d)}) T^{ab[c,d]} \\ &= -\frac{1}{2}(V_\mu^{(a)} V_\nu^{(b)} V^{\mu(c)} V^{\nu(d)} + V_\nu^{(a)} V_\mu^{(b)} V^{\nu(c)} V^{\mu(d)}) T^{ab[c,d]} \\ &= -\frac{1}{2}V_\mu^{(a)} V_\nu^{(b)} V^{\mu(c)} V^{\nu(d)}(T^{ab[c,d]} + T^{ba[d,c]}) = -\frac{1}{2}V_\mu^{(a)} V_\nu^{(b)} V^{\mu(c)} V^{\nu(d)} T^{[a,b][c,d]} \end{aligned} \quad (3.28)$$

Now everything is prepared for writing down the final lagrangian.

3.2.3 The final lagrangian written in component fields

The component expansion of the supersymmetric and gauge invariant lagrangian for a renormalizable nonabelian theory is

$$\begin{aligned} \mathscr{L} &= i\lambda^{(a)}\sigma^\mu\mathscr{D}_\mu\bar{\lambda}^{(a)} - \frac{1}{4}F_{\mu\nu}^{(a)}F^{(a)\ \mu\nu} + \frac{1}{2}D^{(a)}D^{(a)} + i(\bar{\psi}_i\bar{\sigma}^\mu\mathscr{D}_\mu\psi_i) + F_i^* F_i \\ &+ (\mathscr{D}_\mu\phi_i)^*(\mathscr{D}^\mu\phi_i) + i\sqrt{2}gT_{ij}^{(a)}[\phi_i^*(\lambda^{(a)}\psi_j) - (\bar{\lambda}^{(a)}\bar{\psi}_i)\phi_j] + gD^{(a)}T_{ij}^{(a)}\phi_i^*\phi_j \end{aligned} \quad (3.29)$$

where the covariant derivatives and the non-abelian field strength tensor are

$$\begin{aligned}
\mathscr{D}_\mu \bar{\lambda}^{(a)} &= \partial_\mu \bar{\lambda}^{(a)} - g f^{abc} V_\mu^{(b)} \bar{\lambda}^{(c)} \\
F_{\mu\nu}^{(a)} &= \partial_\mu V_\nu^{(a)} - \partial_\nu V_\mu^{(a)} - g f^{abc} V_\mu^{(b)} V_\nu^{(c)} \\
\mathscr{D}_\mu \psi &= \partial_\mu \psi + igV_\mu \psi \\
\mathscr{D}_\mu \phi &= \partial_\mu \phi + igV_\mu \phi
\end{aligned} \tag{3.30}$$

3.3 Implementing $G = U(1)_Y \otimes SU(2)_L \otimes SU(3)_C$ group

Generators of the Lie algebra $\mathcal{G}$ are

$$T = \frac{Y}{2} \otimes 1 \otimes 1 + 1 \otimes \frac{\tau^i}{2} \otimes 1 + 1 \otimes 1 \otimes \frac{\lambda^a}{2} \tag{3.31}$$

We remark that the three parts of the generator T commutes among themselves.

The matrix vector superfield consists of the following three parts

$$2gV \rightarrow 2g'V' + 2gV^i + 2g_s V^a \tag{3.32}$$

Now the $(\theta\theta)(\bar{\theta}\bar{\theta})$-component from the component expansion of $\Phi^\dagger e^{2g'V'+2gV^i+2g_sV^a}\Phi$ will lead to the same terms as in equations (3.19) - (3.24) but with gV_μ replaced by $g'V'_\mu + gV^i_\mu + g_s V^a_\mu$. This leads to the covariant derivatives of the fields ϕ, ψ given by

$$\begin{aligned}
\mathscr{D}_\mu \psi &= \partial_\mu \psi + ig'\frac{1}{2}YV'_\mu \psi + ig\frac{1}{2}\tau^i V^i_\mu \psi + ig_s\frac{1}{2}\lambda^a V^a_\mu \psi \\
\mathscr{D}_\mu \phi &= \partial_\mu \phi + ig'\frac{1}{2}YV'_\mu \phi + ig\frac{1}{2}\tau^i V^i_\mu \phi + ig_s\frac{1}{2}\lambda^a V^a_\mu \phi
\end{aligned} \tag{3.33}$$

The term $e^{-V} D_a e^V$ can be schematically written as

$$e^{-(V_1+V_2+V_3)} D_a e^{(V_1+V_2+V_3)} \tag{3.34}$$

This expression will fall to three pieces because the V_i commutes with V_j $(i \neq j)$ and because the function e^{V_i} contains only terms with even number of Grassman variables which enable us to use the standard Leibnitz rule when making derivation of products of functions. Finally we are left with three identical terms in the lagrangian, one for each group

$$\begin{aligned}
&i\lambda'\sigma^\mu \mathscr{D}_\mu \bar{\lambda}' - \frac{1}{4}F'_{\mu\nu}F'^{\,\mu\nu} + \frac{1}{2}D'D' \\
&i\lambda^i\sigma^\mu \mathscr{D}_\mu \bar{\lambda}^i - \frac{1}{4}F^i_{\mu\nu}F^{i\,\mu\nu} + \frac{1}{2}D^iD^i \\
&i\lambda^a\sigma^\mu \mathscr{D}_\mu \bar{\lambda}^a - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \frac{1}{2}D^aD^a
\end{aligned} \tag{3.35}$$

where each group has its own covariant derivative.

Chapter 4

MSSM theory

4.1 Lagrangian of the MSSM

In the previous chapter we have derived the supersymmetric lagrangian for general scalar, spinor and vector fields. In this chapter we will put the fields that are already known together with their predicted superpartners and suggested two Higgs doublets with their superpartners into the theory. At the end we will derive the couplings that are important by counting the width for the neutralino decay including only QCD corrections.

The MSSM field content is summarized in the following table (Tab. 4.1).

Superfield	Particle	Spin	$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_W \otimes \mathrm{U}(1)_Y$	Superpartner	Spin
$\hat{V}_1$	B_μ	1	(1, 1, 0)	$\tilde{B}$	$\frac{1}{2}$
$\hat{V}_2$	W_μ^i	1	(1, 3, 0)	$\tilde{W}^i$	$\frac{1}{2}$
$\hat{V}_3$	G_μ^a	1	(8, 1, 0)	$\tilde{g}^a$	$\frac{1}{2}$
$\hat{Q}$	$Q = (u_L, d_L)$	$\frac{1}{2}$	$(3, 2, \frac{1}{3})$	$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$	0
$\hat{U}^c$	$U^c = \bar{u}_R$	$\frac{1}{2}$	$(3^*, 1, \text{-}\frac{4}{3})$	$\tilde{U}^c = \tilde{u}_R^*$	0
$\hat{D}^c$	$D^c = \bar{d}_R$	$\frac{1}{2}$	$(3^*, 1, \frac{2}{3})$	$\tilde{D}^c = \tilde{d}_R^*$	0
$\hat{L}$	$L = (\nu_L, e_L)$	$\frac{1}{2}$	(1, 2, -1)	$\tilde{L} = (\tilde{\nu}_L, \tilde{e}_L)$	0
$\hat{E}^c$	$E^c = \bar{e}_R$	$\frac{1}{2}$	(1, 1, 2)	$\tilde{E}^c = \tilde{e}_R^*$	0
$\hat{H}_1$	$H_1 = (H_1^0, H_1^-)$	0	(1, 2, -1)	$\tilde{H}_1 = (\tilde{H}_1^0, \tilde{H}_1^-)$	$\frac{1}{2}$
$\hat{H}_2$	$H_2 = (H_2^+, H_2^0)$	0	(1, 2, 1)	$\tilde{H}_2 = (\tilde{H}_2^+, \tilde{H}_2^0)$	$\frac{1}{2}$

Table 4.1: Particle content of the MSSM.

The names for the new fields are as follows. The superpartners to gauge bosons B_μ, W_μ^i, G_μ^a are called gauginos and carry the spin $\frac{1}{2}$. Gluinos are fermionic superpartners to gluons, the other gauginos combine to form photino, Z-ino and the W-inos. Superpartners are denoted by tilde.

The second thing we want to mention in connection with the table is the bar upon the term e_R, d_R, u_R. This bar indicates that we will substitute the Weyl fermion with undotted indices into the lagrangian. In the appendix we can see that the right-handed part of the Dirac spinor ψ_R is in fact the dotted spinor.

Finally, in supersymmetric theories one needs two complex Higgs doublets to give mass to both the up-type and down-type fermions. This is because the superpotential must consist of the combinations of chiral superfields that are chiral. We cannot simply use the complex conjugate of the Higgs doublet as in the Standard Model otherwise the $\theta\theta$-component of superpotential would not transform into a total divergence.

The lagrangian of the MSSM can be written in the following way [10]

$$\mathscr{L}_{\text{MSSM}} = \mathscr{L}_{\text{kinetic}} - V_Y - V_F - V_D - V_{\tilde{G}\psi\tilde{\psi}} + \mathscr{L}_{\text{soft}} \tag{4.1}$$

where the $\mathscr{L}_{\text{kinetic}}$ contains the standard kinetic terms including gauge interactions with the gauge bosons. The terms $V_Y, V_F, V_D, V_{\tilde{G}\psi\tilde{\psi}}$ stand for all interaction that are allowed in the supersymmetric theory and the $\mathscr{L}_{\text{soft}}$ includes the soft supersymmetry breaking terms.

Superpotential

$$W = -\varepsilon_{ij}\left[h_e \hat{H}_1^i \hat{L}^j \hat{E}^c + h_d \hat{H}_1^i \hat{Q}^j \hat{D}^c + h_u \hat{H}_2^j \hat{Q}^i \hat{U}^c - \mu \hat{H}_1^i \hat{H}_2^j\right] + \text{h.c.} \tag{4.2}$$

The ε_{ij} tensor is still the same tensor we use from the beginning, that is, $\varepsilon_{12} = -1$. By writing the superpotential we have suppressed possible generation indices on superfields. The superpotential give rise to two kind of interactions described by the Yukawa potential V_Y and the so-called F-term potential V_F (see equations (2.28) and (2.29)).

The Yukawa potential

The Yukawa potential is obtained by substituting two of the superfields by their fermionic content and the remaining superfield (if something remains) by its scalar content. The result is

$$\begin{aligned} V_Y &= -\varepsilon_{ij}\left[h_e H_1^i L^j E^c + h_d H_1^i Q^j D^c + h_u H_2^j Q^i U^c - \mu \tilde{H}_1^i \tilde{H}_2^j\right] \\ &- \varepsilon_{ij}\left[h_e \tilde{H}_1^i L^j \tilde{E}^c + h_d \tilde{H}_1^i Q^j \tilde{D}^c + h_u \tilde{H}_2^j Q^i \tilde{U}^c\right] \\ &- \varepsilon_{ij}\left[h_e \tilde{H}_1^i \tilde{L}^j E^c + h_d \tilde{H}_1^i \tilde{Q}^j D^c + h_u \tilde{H}_2^j \tilde{Q}^i U^c\right] + \text{h.c.} \end{aligned} \tag{4.3}$$

The F-term potential

The F-term potential arises after using the Lagrange-Euler equations of motion for the auxiliary field F and substituting them back into the lagrangian. We will demonstrate this later when discussing the particle spectrum of the MSSM theory. The F-term superpotential is then given by

$$V_F = \sum_i \left| \frac{\partial W(\phi)}{\partial \phi_i} \right|^2 \tag{4.4}$$

where ϕ_i are the scalar components of the superfields.

The D-term potential

This potential comes, analogously as in the case of F-term potential, from eliminating the auxiliary field D using equation of motion and substituting back into the lagrangian. The D-term potential is given by

$$V_D = \frac{1}{2} \sum_a D^a D^a \tag{4.5}$$

where

$$D^a = -g^a \phi_i^* T_{ij}^{(a)} \phi_j \tag{4.6}$$

The ϕ_i are the scalar components of the superfields and the $T^{(a)}((a) \leftrightarrow \ ', i, a)$ are the generators of the particular gauge symmetry.

The $V_{\tilde{G}\psi\tilde{\psi}}$ potential

This potential comes from the equations (3.22) and (3.23). It represents an interaction of gauginos.

$$V_{\tilde{G}\psi\tilde{\psi}} = i\sqrt{2} g T_{ij}^{(a)} (\bar{\lambda}^{(a)} \bar{\psi}_i) \phi_j - i\sqrt{2} g T_{ij}^{(a)} \phi_i^* (\lambda^{(a)} \psi_j) \tag{4.7}$$

where ϕ, ψ are the scalar, resp. fermionic components of the chiral superfield and $\lambda^{(a)}$ is the gaugino field.

The last term in the full MSSM-lagrangian includes soft supersymmetric breaking terms. The word soft means that we do not consider any dimensionless SUSY-breaking couplings.

$$\begin{aligned} -\mathscr{L}_{\text{soft}} &= m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 - m_{12}^2 \varepsilon_{ij} (H_1^i H_2^j + H_1^{\dagger i} H_2^{\dagger j}) + \frac{1}{2} m_{\tilde{g}} \tilde{g}^a \tilde{g}^a + \frac{1}{2} M \tilde{W}^i \tilde{W}^i \\ &+ \frac{1}{2} M' \tilde{B} \tilde{B} + M_{\tilde{Q}}^2 |\tilde{q}_L|^2 + M_{\tilde{U}}^2 |\tilde{u}_R^c|^2 + M_{\tilde{D}}^2 |\tilde{d}_R^c|^2 + M_{\tilde{L}}^2 |\tilde{l}_L|^2 + M_{\tilde{E}}^2 |\tilde{e}_R^c|^2 \\ &- \varepsilon_{ij} \Big(h_e A_e H_1^i \tilde{L}^j \tilde{E}^c + h_d A_d H_1^i \tilde{Q}^j \tilde{D}^c + h_u A_u H_2^j \tilde{Q}^i \tilde{U}^c + \text{h.c.} \Big) \end{aligned} \tag{4.8}$$

where we have introduced the SUSY-breaking mass parameters $m_{H_1}^2, m_{H_2}^2, m_{12}^2, m_{\tilde{g}}, M,$ $M', M_{\tilde{Q}}^2, M_{\tilde{U}}^2, M_{\tilde{D}}^2, M_{\tilde{L}}^2, M_{\tilde{E}}^2$ as well as the SUSY-breaking trilinear scalar couplings $A_e, A_u,$ A_d.

4.2 Particle spectrum of the MSSM

4.2.1 Higgs sector

The MSSM theory needs two Higgs doublets. Their hypercharges are: $Y_{H_1} = -1, Y_{H_2} = 1$. The doublet H_2 is responsible for masses of the up-type fermions and the doublet H_1 for masses of the down-type fermions.

The potential for the Higgs field after expanding the lagrangian is

$$\begin{aligned} V &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_{12}^2\varepsilon_{ij}(H_1^i H_2^j + H_1^{\dagger i}H_2^{\dagger j}) \\ &+ \frac{1}{8}(g^2+g'^2)(|H_1|^2-|H_2|^2)^2 + \frac{1}{2}g^2|H_1^\dagger H_2|^2 \end{aligned} \tag{4.9}$$

where $m_{1,2}^2 = m_{H_{1,2}}^2 + |\mu|^2$.

Now we will show the derivation of the terms containing the constant μ. The relevant terms are

$$\begin{aligned} W &\leftrightarrow \varepsilon_{ij}\,\mu\,(H_1^i F_{H_2}^j + F_{H_1}^i H_2^j) + \text{h.c.} = \\ &= -\mu H_1^0 F_{H_2^0} + \mu H_1^- F_{H_2^+} - \mu F_{H_1^0} H_2^0 + \mu F_{H_1^-} H_2^+ \\ F_i^* F_i|_{\mathscr{L}} &\leftrightarrow F_{H_1^0}^* F_{H_1^0} + F_{H_1^-}^* F_{H_1^-} + F_{H_2^+}^* F_{H_2^+} + F_{H_2^0}^* F_{H_2^0} \end{aligned} \tag{4.10}$$

After using the Euler-Lagrange equation of motion for $F_{H_1^-}, F_{H_1^-}^*$ we get

$$F_{H_1^-}^* = -\mu H_2^+ + \ldots, \quad F_{H_1^-} = -\mu^* H_2^{+*} + \ldots \tag{4.11}$$

The dots indicate that the previous two equations are incomplete. There is another term in the superpotential that contains the $F_{H_1^-}$ field as well as the fields associated with the electron and therefore has no contribution to the Higgs potential.
Substituting back we obtain the following term to $\mathscr{L}_{\text{MSSM}}$

$$-F_{H_1^-} F_{H_1^-}^* = -|\mu|^2|H_2^+|^2 + \ldots \tag{4.12}$$

Analogously we obtain another terms $-|\mu|^2|H_2^0|^2, -|\mu|^2|H_1^0|^2 \text{ and } -|\mu|^2|H_1^-|^2$.

Next we will derive the terms containing the gauge couplings g, g'. The relevant terms that come from the final lagrangian (3.29) are:

$$\begin{aligned} &\frac{1}{2}D_B D_B + \frac{1}{2}D_{W^i}D_{W^i} + g' D_B (H_1)_a^* \left(\frac{Y}{2}\right)_{ab}(H_1)_b + g' D_B (H_2)_a^* \left(\frac{Y}{2}\right)_{ab}(H_2)_b \\ &+ g\sum_i D_{W^i}(H_1)_a^* \left(\frac{\tau^i}{2}\right)_{ab}(H_1)_b + g\sum_i D_{W^i}(H_2)_a^* \left(\frac{\tau^i}{2}\right)_{ab}(H_2)_b \end{aligned} \tag{4.13}$$

The E-L equation of motion for the field D_B is

$$D_B = \frac{g'}{2}(|H_1|^2 - |H_2|^2) + \ldots \tag{4.14}$$

And after substituting back to final lagrangian we obtain the following term to $\mathscr{L}_{\text{MSSM}}$

$$-\frac{1}{2}D_B D_B = -\frac{1}{8}g'^2(|H_1|^2 - |H_2|^2)^2 + \ldots \tag{4.15}$$

The E-L equation of motion for the field D_{W^i} has the following form

$$D_{W^i} = -g\left((H_1^*)_a \left(\frac{\tau^i}{2}\right)_{ab} (H_1)_b + (H_2^*)_a \left(\frac{\tau^i}{2}\right)_{ab} (H_2)_b\right) + \dots \tag{4.16}$$

And after substituting back to final lagrangian we obtain the following term to $\mathscr{L}_{\text{MSSM}}$

$$-\frac{1}{2} D_{W^i} D_{W^i} = -\frac{1}{2} g^2 |H_1^\dagger H_2|^2 - \frac{1}{8} g^2 (|H_1|^2 - |H_2|^2)^2 + \dots \tag{4.17}$$

where we have used the identity $\sum_i (\frac{\tau^i}{2})_{ab} (\frac{\tau^i}{2})_{cd} = \frac{1}{2}(\delta_{ad}\delta_{bc} - \frac{1}{2}\delta_{ab}\delta_{cd})$

Both neutral Higgs boson fields acquire a non-vanishing vacuum expectation value (VEV)

$$\langle H_1 \rangle = \begin{pmatrix} \frac{v_1}{\sqrt{2}} \\ 0 \end{pmatrix}, \qquad \langle H_2 \rangle = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix} \tag{4.18}$$

We will parametrize the doublets in the following way

$$H_1 \equiv \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} (v_1 + \phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}, \qquad Y_{H_1} = -1 \tag{4.19}$$

$$H_2 \equiv \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ (v_2 + \phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}, \qquad Y_{H_2} = +1 \tag{4.20}$$

The gauge bosons are made massive after electro-weak symmetry breaking. Because their masses are functions of v_1, v_2, their experimentally measured values can fix one of the VEVs as can be seen from relations

$$m_Z^2 = \frac{g^2 + g'^2}{4}(v_1^2 + v_2^2), \qquad m_W^2 = \frac{g^2}{4}(v_1^2 + v_2^2) \tag{4.21}$$

$$v^2 \equiv (v_1^2 + v_2^2) = \frac{4m_Z^2}{g^2 + g'^2} \approx (246\text{GeV})^2 \tag{4.22}$$

The other VEV remains a free parameter of the theory. Conventionally, people do not work with this parameter but introduce the angle β which is defined as

$$\tan\beta \equiv \frac{v_2}{v_1} \geq 0, \qquad 0 \leq \beta \leq \frac{\pi}{2} \tag{4.23}$$

However, $\tan\beta$ is not entirely a free parameter. The conditions for the minimum of the Higgs potential

$$\left.\frac{\partial V}{\partial H_1^0}\right|_{\langle H_n^0 \rangle = v_n} = \left.\frac{\partial V}{\partial H_2^0}\right|_{\langle H_n^0 \rangle = v_n} = 0 \tag{4.24}$$

restrict the parameters $\tan\beta, m_{H_1}^2, m_{H_2}^2, |\mu|^2$ and m_{12}^2 by the following equations

$$m_1^2 = -m_{12}^2 \tan\beta - \frac{1}{2} m_Z^2 \cos 2\beta \tag{4.25}$$

$$m_2^2 = -m_{12}^2 \cot\beta + \frac{1}{2} m_Z^2 \cos 2\beta \tag{4.26}$$

The Higgs mass matrix can generally be written as

$$M_{ij}^{2,\mathrm{Higgs}} = \frac{1}{2}\frac{\partial^2 V}{\partial H_i \partial H_j}\bigg|_{\langle H_n^0\rangle = v_n} \tag{4.27}$$

This mass matrix has the block-diagonal form. The particular 2×2 blocks can separately be diagonalized as

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \tag{4.28}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix} \tag{4.29}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} \tag{4.30}$$

The $G^0, G^\pm$ are massless Goldstone bosons, H^0, h^0, A^0 are three neutral Higgs bosons, and $H^\pm$ are two charged Higgs bosons. The three free parameters of the Higgs sector are conventionally chosen to be

$$m_{A^0}, \qquad \tan\beta, \qquad \mu \tag{4.31}$$

The other parameters of the Higgs sector expressed with the help of these three parameters are

$$m_{h^0,H_0}^2 = \frac{1}{2}\left[m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2+m_Z^2)^2 - 4m_{A^0}^2 m_Z^2 \cos^2\beta}\right] \tag{4.32}$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 \tag{4.33}$$

$$\tan 2\alpha = \tan 2\beta \frac{m_{A^0}^2 + m_Z^2}{m_{A^0}^2 - m_Z^2} \tag{4.34}$$

4.2.2 The SM fermions

Our aim is to obtain the Dirac lagrangian for fermions known from the Standard Model. As an example we will derive the desired lagrangian for electrons. There are two superfields at our disposal, superfield $\hat{L}$ and the superfield $\hat{E}^c$. When we substitute their fermionic content ($e_L, \bar{e}_R$ - both are Weyl spinors with undotted indices) into the final lagrangian (3.29) instead of ψ we obtain the following two terms

$$i\bar{e}_L\bar{\sigma}^\mu \mathscr{D}_\mu e_L + i\bar{\bar{e}}_R\bar{\sigma}^\mu \mathscr{D}_\mu \bar{e}_R = i\bar{e}_L\bar{\sigma}^\mu \mathscr{D}_\mu e_L + i\bar{e}_R\sigma^\mu \mathscr{D}_\mu e_R \tag{4.35}$$

which is the massless part of the Dirac equation that agrees with the Peskin-Schroeder notation. The mass term of the electron appears after the Higgs bosons get their VEVs

$$\mathscr{L} \leftrightarrow -V_Y \rightarrow \varepsilon_{ij} h_e H_1^i L^j E^c + \text{h.c.} = -h_e \frac{v_1}{\sqrt{2}}(e_L\bar{e}_R + \bar{e}_L e_R) \tag{4.36}$$

The mass term is of correct sign provided that the constants h_e, v_1 are positive.

4.2.3 Gauginos

In this subsection we will treat as an example the gluinos case. Our aim is to check if the terms containing gluinos in our lagrangian agree with the terms given by Majorana lagrangain. We said Majorana because there are no two superfields as was the case of the electrons. We remark that gauge bosons are themselves antiparticles and the same applies for their superpartners - gauginos. The gluino written as a Majorana spinor is

$$\tilde{g}^a = \begin{pmatrix} -i\lambda^a \\ i\bar{\lambda}^a \end{pmatrix}, \qquad a = 1, 2 \ldots 8 \tag{4.37}$$

Then the Majorana lagrangian yields

$$\mathscr{L}_M = \frac{i}{2}\bar{\tilde{g}}^a \gamma^\mu \mathscr{D}_\mu \tilde{g}^a - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}}^a \tilde{g}^a = i\lambda^a \sigma^\mu \mathscr{D}_\mu \bar{\lambda}^a + \frac{1}{2} m_{\tilde{g}} (\lambda^a \lambda^a + \bar{\lambda}^a \bar{\lambda}^a) \tag{4.38}$$

The first term agrees with the first term in the final lagrangian (3.29). The second term agrees with the soft breaking term in the $\mathscr{L}_{\text{soft}}$.

4.2.4 Neutralino sector

The fermionic superpartners of the gauge bosons (gauginos) and the superpartners of the Higgs bosons (higgsinos) mix to form mass eigenstates called neutralinos (particles with zero charge) and charginos (charged particles).

In the interaction base on can combine the four neutral Weyl states as

$$\psi_j^0 = (\tilde{B}, \tilde{W}_3^0, \tilde{H}_1^0, \tilde{H}_2^0) \tag{4.39}$$

where the $\tilde{B}, \tilde{W}_3^0 \leftrightarrow -i\lambda$. The mass lagrangian written in terms of the vector ψ^0 is

$$\mathscr{L} = -\frac{1}{2}(\psi^0)^T Y \psi^0 + \text{h.c.} \tag{4.40}$$

where the neutralino mass matrix Y is

$$Y = \begin{pmatrix} M' & 0 & -m_Z s_W \cos\beta & m_Z s_W \sin\beta \\ 0 & M & m_Z c_W \cos\beta & -m_Z c_W \sin\beta \\ -m_Z s_W \cos\beta & m_Z c_W \cos\beta & 0 & -\mu \\ m_Z s_W \sin\beta & -m_Z c_W \sin\beta & -\mu & 0 \end{pmatrix} \tag{4.41}$$

The letter W stands for the Weinberg angle and the s_W, c_W are short forms of the sine and cosine of this angle. We now get little back and expand the lagrangian (4.40). Its expanded form yields following terms

$$\begin{aligned} \mathscr{L} &= -\frac{1}{2}M'\tilde{B}\tilde{B} - \frac{1}{2}M\tilde{W}_3^0\tilde{W}_3^0 + \mu\tilde{H}_1^0\tilde{H}_2^0 - m_Z\, c_W \cos\beta\, \tilde{W}_3^0\tilde{H}_1^0 \\ &+ m_Z\, c_W \sin\beta\, \tilde{W}_3^0\tilde{H}_2^0 + m_Z\, s_W \cos\beta\, \tilde{B}\tilde{H}_1^0 - m_Z\, s_W \sin\beta\, \tilde{B}\tilde{H}_2^0 \end{aligned} \tag{4.42}$$

The first two are from $\mathscr{L}_{\text{soft}}$, the third comes from the Yukawa potential V_Y and the rest come from the potential $V_{\tilde{G}\psi\tilde{\psi}}$ where for example

$$\begin{aligned}\mathscr{L} \leftrightarrow -V_{\tilde{G}\psi\tilde{\psi}} \quad : \quad & i\sqrt{2}g'\left[\frac{v_1^*}{\sqrt{2}}\left(-\frac{1}{2}\right)\lambda_{\tilde{B}}\tilde{H}_1^0 + \frac{v_2^*}{\sqrt{2}}\left(\frac{1}{2}\right)\lambda_{\tilde{B}}\tilde{H}_2^0\right] + \dots \\ & = -im_Z\, s_W \cos\beta\,\lambda_{\tilde{B}}\tilde{H}_1^0 + im_Z\, s_W \sin\beta\,\lambda_{\tilde{B}}\tilde{H}_2^0 + \dots \end{aligned} \tag{4.43}$$

Due to the Majorana nature of the neutralinos, the mass matrix can be diagonalised using only one rotation matrix Z

$$ZYZ^{-1} = \mathrm{diag}\,(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}), \qquad |m_{\tilde{\chi}_1^0}| \le |m_{\tilde{\chi}_2^0}| \le |m_{\tilde{\chi}_3^0}| \le |m_{\tilde{\chi}_4^0}| \tag{4.44}$$

where we assume that the mixing matrix is real and we also allow the eigenvalues to be negative.
The 4-component Majorana spinors for the neutralino fields can be constructed as

$$\tilde{\chi}_i^0 \equiv Z_{ij}\begin{pmatrix} \psi_j^0 \\ \bar{\psi}_j^0 \end{pmatrix} \tag{4.45}$$

4.2.5 Chargino sector

The superpartners of the charged gauge bosons and charged Higgs bosons mix to create charginos. In the Weyl representation we have

$$\psi^+ = (\tilde{W}^+, \tilde{H}_2^+) \qquad \psi^- = (\tilde{W}^-, \tilde{H}_1^-) \tag{4.46}$$

where $W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp W^2)$ and $W^\pm \leftrightarrow -i\lambda^\pm$. The mass lagrangian in this basis is

$$\mathscr{L} = -\frac{1}{2}(\psi^+, \psi^-)\begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{h.c} \tag{4.47}$$

where the chargino mass matrix is

$$X = \begin{pmatrix} M & \sqrt{2}m_W \sin\beta \\ \sqrt{2}m_W \cos\beta & \mu \end{pmatrix} \tag{4.48}$$

This matrix can be diagonalized by using two unitary matrices U and V.

$$UXV^{-1} = \mathrm{diag}\,(m_{\chi_1^\pm}, m_{\chi_2^\pm}), \qquad |m_{\chi_1^\pm}| \le |m_{\chi_2^\pm}| \tag{4.49}$$

We use a convention in which the matrices U, V are real. It implies that the eigenvalues can be negative. The Dirac spinor is constructed as

$$\tilde{\chi}_i^+ \equiv \begin{pmatrix} V_{ij}\,\psi_j^+ \\ U_{ij}\,\bar{\psi}_j^- \end{pmatrix} \tag{4.50}$$

The mass eigenvalues are given by

$$m^2_{\tilde{\chi}^\pm_{1,2}} = \frac{1}{2}\left[M^2 + \mu^2 + 2m_W^2 \mp \sqrt{(M^2 + \mu^2 + 2m_W^2)^2 - 4(m_W^2 \sin 2\beta - \mu M)^2}\right] \tag{4.51}$$

4.2.6 Sfermion sector

The sfermion mass matrix has its origin in the F-term, D-term potentials, SUSY breaking potential and in the trilinear couplings where the neutral Higgs fields get their VEVs. As an example we will derive the mass matrix for selectrons. Finally we will write down the general mass matrix for all sfermions, squarks including.

The terms that contribute to the selectron mass matrix are

$$\mathscr{L}_{\text{soft}}|_{\text{mass terms}} : -M^2_{\tilde{L}}|\tilde{l}_L|^2 - M^2_{\tilde{E}}|\tilde{e}^c_R|^2 \tag{4.52}$$

$$\mathscr{L}_{\text{soft}}|_{\text{tril. coup.}} : \varepsilon_{ij} h_e A_e H^i_1 \tilde{L}^j \tilde{E}^c + \text{h.c} = -h_e A_e \frac{v_1}{\sqrt{2}} \tilde{e}_L \tilde{e}^*_R + \text{h.c} \tag{4.53}$$

$$F-\text{term:} - \varepsilon_{ij}(h_e H^i_1 L^j E^c - \mu H^i_1 H^j_2) \tag{4.54}$$

$$\rightarrow -|h_e|^2 \frac{v_1^2}{2}(|e_L|^2 + |e_R|^2) + h_e \mu \frac{v_2}{\sqrt{2}} \tilde{e}_L \tilde{e}^*_R + \text{h.c} \tag{4.55}$$

$$D-\text{term:} - \frac{1}{8} g^2 \Big[\tilde{L}^*_a \tau^i_{ab} \tilde{L}_b + (H_1)^*_a \tau^i_{ab} (H_1)_b + (H_2)^*_a \tau^i_{ab} (H_2)_b + \dots\Big]^2$$

$$\rightarrow -\frac{1}{8} g^2 \Big[- \tilde{e}^*_L \tilde{e}_L + \frac{v_1^2}{2} - \frac{v_2^2}{2} + \dots\Big]^2 \tag{4.56}$$

$$-\frac{1}{8} g'^2 \Big[\tilde{e}^*_L Y \tilde{e_L} + \tilde{e}_R Y \tilde{e}^*_R + H^0_1{}^* Y H^0_1 + H^0_2{}^* Y H^0_2 + \dots\Big]^2$$

$$\rightarrow -\frac{1}{8} g'^2 \Big[- |\tilde{e}_L|^2 + 2|\tilde{e}_R|^2 - \frac{v_1^2}{2} + \frac{v_2^2}{2} + \dots\Big]^2 \tag{4.57}$$

We suppose that the parameters μ, h_e are real. Now it is not complicated to make up the selectron mass matrix with the help of previous equations. However, we will not do that but write down the general form for the sfermion mass matrix instead and the agreement in the selectron case can be easily verified

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_L} & a_f m_f \\ a_f m_f & m^2_{\tilde{f}_R} \end{pmatrix} \tag{4.58}$$

where

$$m^2_{\tilde{f}_L} = M^2_{\{\tilde{Q},\tilde{L}\}} + (I^{3L}_f - e_f s^2_W) \cos 2\beta m^2_Z + m^2_f \tag{4.59}$$

$$m^2_{\tilde{f}_R} = M^2_{\{\tilde{U},\tilde{D},\tilde{E}\}} + e_f s^2_W \cos 2\beta m^2_Z + m^2_f \tag{4.60}$$

$$a_f = A_f - \mu (\tan\beta)^{-2I^{3L}_f} \tag{4.61}$$

The term I^{3L}_f denotes the third component of the weak isospin of the fermion, e_f denotes the electric charge in terms of the elementary charge e. The other terms were presented earlier.

We have to diagonalize the matrix in order to obtain the mass eigenstates. We introduce the mixing angle $\theta_{\tilde{f}}$. The diagonalization proceeds as follows

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_L} & a_f m_f \\ a_f m_f & m^2_{\tilde{f}_R} \end{pmatrix} = \left(R^{\tilde{f}}\right)^\dagger \begin{pmatrix} m^2_{\tilde{f}_1} & 0 \\ 0 & m^2_{\tilde{f}_2} \end{pmatrix} \left(R^{\tilde{f}}\right) \tag{4.62}$$

where

$$\left(R^{\tilde f}\right) = \begin{pmatrix} \cos\theta_{\tilde f} & \sin\theta_{\tilde f} \\ -\sin\theta_{\tilde f} & \cos\theta_{\tilde f} \end{pmatrix} \tag{4.63}$$

The relation between the mass eigenstates $\tilde f_i$ and the interaction eigenstates $\tilde f_\alpha$ is

$$\tilde f_i = \begin{pmatrix} \tilde f_1 \\ \tilde f_2 \end{pmatrix} = R^{\tilde f}.\begin{pmatrix} \tilde f_L \\ \tilde f_R \end{pmatrix} = R^{\tilde f}_{i\alpha}\tilde f_\alpha, \qquad \tilde f_\alpha = \begin{pmatrix} \tilde f_L \\ \tilde f_R \end{pmatrix} = (R^{\tilde f})^T.\begin{pmatrix} \tilde f_1 \\ \tilde f_2 \end{pmatrix} = R^{\tilde f}_{i\alpha}\tilde f_i \tag{4.64}$$

The mass eigenvalues and the mixing angle are

$$m^2_{\tilde f_{1,2}} = \frac{1}{2}\left(m^2_{\tilde f_L} + m^2_{\tilde f_R} \mp \sqrt{(m^2_{\tilde f_L} - m^2_{\tilde f_R})^2 + 4a_f^2 m_f^2}\right) \tag{4.65}$$

$$\cos\theta_{\tilde f} = \frac{-a_f m_f}{\sqrt{(m^2_{\tilde f_L} - m^2_{\tilde f_1})^2 + a_f^2 m_f^2}} \qquad (0 \le \theta_{\tilde f} < \pi) \tag{4.66}$$

4.3 Couplings important for the neutralino decay

4.3.1 Neutralino-Fermion-Sfermion couplings

The relevant lagrangian comes from the two potentials, the Yukawa potential V_Y and the potential $V_{\tilde G\psi\tilde\psi}$. Here are as an example two terms that arise from this potentials for the case of sbottom and bottom quark

$$\begin{aligned} \mathscr{L} &\leftrightarrow -V_Y : -h_b \tilde H_1^0 b_L \tilde b_R^* + \ldots = -\bar{\tilde\chi}_k^0 h_b Z_{k3} R^{\tilde f}_{i2} P_L b \tilde b_i^* + \ldots \\ \mathscr{L} &\leftrightarrow -V_{\tilde G\psi\tilde\psi} : -\sqrt{2}g'\bar{\tilde B}\bar b_L \frac{Y}{2}\tilde b_L + \ldots = -\sqrt{2}g\tan\theta_W \bar b(e_b - I_b^{3L})R^{\tilde f}_{i1} P_R \tilde\chi_k^0 \tilde f_i \end{aligned} \tag{4.67}$$

For the neutralino-fermion-sfermion couplings the whole lagrangian reads

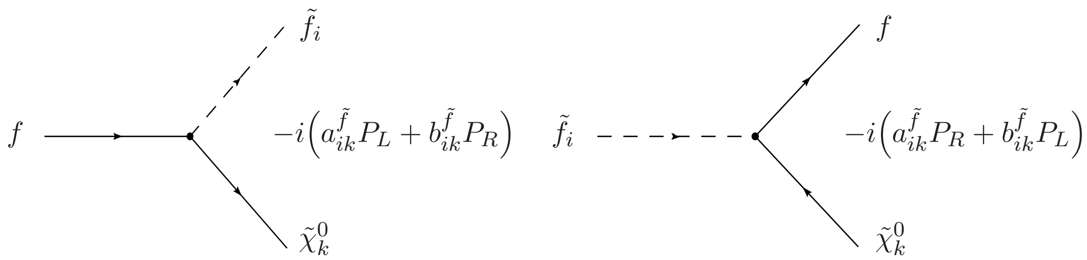
$$\mathscr{L} = -\bar f\left(a^{\tilde f}_{ik}P_R + b^{\tilde f}_{ik}P_L\right)\tilde\chi_k^0 \tilde f_i - \bar{\tilde\chi}_k^0\left(a^{\tilde f}_{ik}P_L + b^{\tilde f}_{ik}P_R\right) f \tilde f_i^* \tag{4.68}$$

where

$$a^{\tilde f}_{ik} = h_f Z_{kx} R^{\tilde f}_{i2} + g f^f_{Lk} R^{\tilde f}_{i1}, \qquad b^{\tilde f}_{ik} = h_f Z_{kx} R^{\tilde f}_{i1} + g f^f_{Rk} R^{\tilde f}_{i2} \tag{4.69}$$

$$f^f_{Lk} = \sqrt{2}((e_f - I_f^{3L})\tan\theta_{\rm W} Z_{k1} + I_f^{3L} Z_{k2}), \qquad f^f_{Rk} = -\sqrt{2}e_f \tan\theta_{\rm W} Z_{k1} \tag{4.70}$$

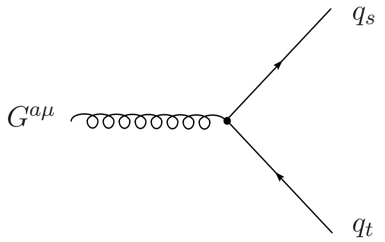
where x takes the values {3, 4} for {down, up} - type case, respectively



4.3.2 Gluon-Fermion-Fermion coupling

The relevant lagrangian for the gluon - quark coupling is

$$\mathscr{L} = -g_s T^a_{st} G^a_\mu \bar{q}_s \gamma^\mu q_t \tag{4.71}$$



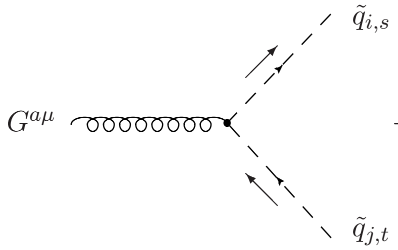
$$-ig_s T^a_{st}\gamma_\mu$$

4.3.3 Gluon-Sfermion-Sfermion coupling

The lagrangian for the gluon - quark coupling comes from the term $(\mathscr{D}_\mu \phi_i)^\dagger(\mathscr{D}^\mu \phi_i)$

$$\mathscr{L} = -ig_s G^a_\mu T^a_{st} \tilde{q}^*_{i,s} \overleftrightarrow{\partial^\mu} \tilde{q}_{i,t} \tag{4.72}$$

where the $\overleftrightarrow{\partial^\mu}$ is defined by: $A \overleftrightarrow{\partial^\mu} B = A(\partial^\mu B) - (\partial^\mu A)B$



$$-ig_s T^a_{st}(p_1 + p_2)\delta_{ij}$$

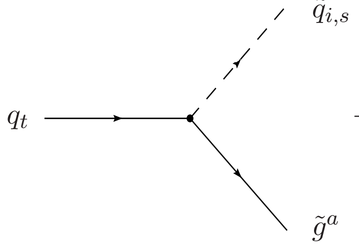
The interaction of the squark with the gluon does not include mixing of the scalar particles.

4.3.4 Gluino-Fermion-Sfermion coupling

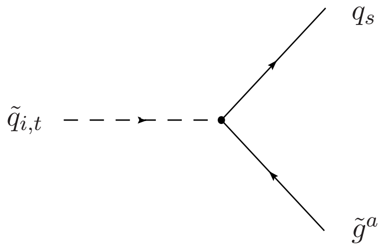
The lagrangian for this coupling comes from the term $-V_{\tilde{G}\psi\tilde{\psi}}$

$$\begin{aligned} \mathscr{L} &= -\sqrt{2} g_s T^a_{st}[(\bar{q}_s P_R \tilde{g}^a \tilde{q}_{L,t} - \bar{q}_s P_L \tilde{g}^a \tilde{q}_{R,t}) + (\bar{\tilde{g}}^a P_L q_t \tilde{q}^*_{L,s} - \bar{\tilde{g}}^a P_R q_t \tilde{q}^*_{R,s})] \\ &= -\sqrt{2} g_s T^a_{st}[\bar{q}_s(R^{\tilde{q}}_{iL} P_R - R^{\tilde{q}}_{iR} P_L)\tilde{g}^a \tilde{q}_{i,t} + \bar{\tilde{g}}^a (R^{\tilde{q}}_{iL} P_L - R^{\tilde{q}}_{iR} P_R) q_t \tilde{q}^*_{i,s} \end{aligned} \tag{4.73}$$

The relative minus sign comes from the fact that the field q_R belongs to conjugate representation, that means, is $SU(3)$ antitriplet.



$$-\sqrt{2} i g_s T^a_{st}(R^{\tilde{q}}_{iL} P_L - R^{\tilde{q}}_{iR} P_R)$$



$$-\sqrt{2} i g_s T^a_{st}(R^{\tilde{q}}_{iL} P_R - R^{\tilde{q}}_{iR} P_L)$$

The arrow on the gluino line indicates whether the gluino participates on the vertex as $\bar{\tilde{g}}^a$ (out) or as $\tilde{g}^a$ (in).

4.3.5 Four Sfermions coupling

In this subsection we will focus on the coupling that contains the strong coupling constant g_s. It suffices then to consider only the D-term potential with

$$D^a = g_s \Big(\tilde{t}^*_{L,s} T^a_{st} \tilde{t}_{L,t} + \tilde{b}^*_{L,s} T^a_{st} \tilde{b}_{L,t} - \tilde{t}_{R,s} T^{a*}_{st} \tilde{t}^*_{R,t} - \tilde{b}_{R,s} T^{a*}_{st} \tilde{b}^*_{R,t} \Big) \tag{4.74}$$

The minus sign appears because the superfields $\tilde{U}, \tilde{D}$ belong to conjugate representation $\bar{3}$ and therefore transform with $-T^{a*}_{st}$. We remark that we now consider only the third generation of quarks.
When we define the following matrix

$$A^\alpha_{ij} = R^\alpha_{i1} R^\alpha_{j1} - R^\alpha_{i2} R^\alpha_{j2} = \begin{pmatrix} \cos 2\theta_{\tilde{q}_\alpha} & -\sin 2\theta_{\tilde{q}_\alpha} \\ -\sin 2\theta_{\tilde{q}_\alpha} & -\cos 2\theta_{\tilde{q}_\alpha} \end{pmatrix} \tag{4.75}$$

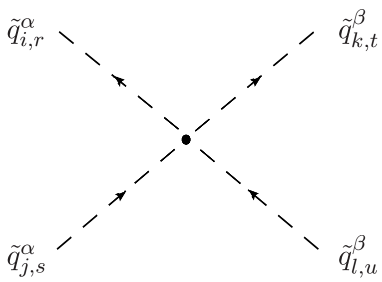
the D^a can be written in a more compact way

$$D^a = g_s T^a_{st} \sum_{\alpha=1,2} A^\alpha_{ij} \tilde{q}^{\alpha *}_{i,s} \tilde{q}^\alpha_{j,t} \tag{4.76}$$

where α = (1,2) corresponds to (Stopsector, Sbottomsector).

The relevant lagrangian for the four sfermions coupling has then the following form

$$\mathscr{L} = -\frac{1}{2} g_s^2 T^a_{rs} T^a_{tu} \sum_{\alpha,\beta=1,2} A^\alpha_{ij} A^\beta_{kl} \tilde{q}^{\alpha *}_{i,r} \tilde{q}^\alpha_{j,s} \tilde{q}^{\beta *}_{k,t} \tilde{q}^\beta_{l,u} \tag{4.77}$$



$$-i g_s^2 \Big(T^a_{rs} T^a_{tu} A^\alpha_{ij} A^\beta_{kl} + \delta_{\alpha\beta} T^a_{ru} T^a_{ts} A^\alpha_{il} A^\alpha_{kj} \Big)$$

where we sum over the index a but not over the index α.

Chapter 5

Renormalization of the MSSM

5.1 Dimensional regularization and reduction

When we want to calculate processes in Quantum field theory at higher than tree level we usually encounter divergencies. Then it is inevitable to regularise the divergent parts and after the renormalization of the theory the formal infinite parts are subtracted.

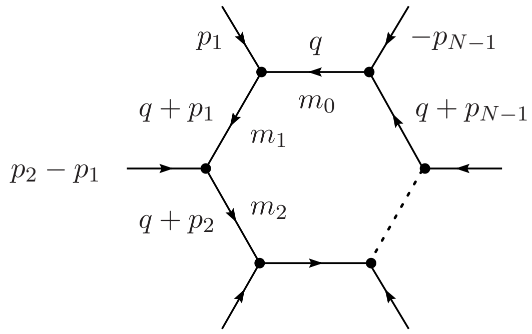
There are two types of divergencies. The first one is called an infrared divergence (IR). It arises as soon as the massless particle appears in the loop. In this work it will be gluon. The way how to tackle the problem is to introduce a small nonzero gluon mass. Then after calculating additional graphs that represent the radiation of the real gluon the final result will be independent from gluon mass and therefore IR convergent.

The second type of divergence is called ultraviolet (UV). It is caused by the divergent behaviour of the loop integrals as the integration variable approaches infinity. The simplest way how to get rid of the infinity provides the so-called cut-off scheme. This method introduces a cut Λ on the energy. With this technique are the Feynman amplitudes finite but the theory looses its Poincaré invariance. Better way was introduced by 't Hooft and Veltman [11]. They realized that by lowering the dimension of an initially divergent integral it can be made finite. We called this method dimensional regularization (DREG). Everything is calculated in D-dimensional space where $D = 4 - 2\varepsilon$ is a complex number. Consequently, the divergent parts arises as a pole of the dimensional parameter D at $\varepsilon = 0$. The whole procedure is described in [11], [12], [13].

A general one loop integral can be written as

$$T^N_{\mu_1 \ldots \mu_M}(p_1, ..., p_{N-1}, m_0, ..., m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{q_{\mu_1} \cdots q_{\mu_M}}{[q^2 - m_0^2 + i\varepsilon][(q + p_1)^2 - m_1^2 + i\varepsilon] ... [(q + p_{N-1})^2 - m_{N-1}^2 + i\varepsilon]} \tag{5.1}$$

where the convention for the momenta are shown in the following picture. The parameter μ serves for retaining the initial dimensionality of the integral.



According to the number of particles in a loop we differentiate the integrals of type A, B, C, D and higher. The first three scalar integrals are denoted as

$$T^1 \equiv A_0(m_0^2) \tag{5.2}$$
$$T^2 \equiv B_0(p_1^2, m_0^2, m_1^2) \tag{5.3}$$
$$T^3 \equiv C_0(p_1^2, (p_1-p_2)^2, p_2^2, m_0^2, m_1^2, m_2^2) \tag{5.4}$$

The other tensor integrals $B^\mu, B^{\mu\nu}, C^\mu, C^{\mu\nu}$ etc. can be calculated from the scalar ones trough the procedure called tensor reduction. We refer to [14]. The divergence is contained in the parameter Δ which is defined as

$$\Delta = \frac{1}{\varepsilon} - \gamma_E + \ln 4\pi \tag{5.5}$$

where $\gamma_E = 0.57721$ is the known Euler-Mascheroni constant. The UV-divergent parts of the loop integrals are listed in the following Table 5.1. We remark that only incomplete (but sufficient for our calculations) set is presented.

Integral		UV divergent part
$A_0(m^2)$	$\rightarrow$	$m^2\Delta$
B_0	$\rightarrow$	Δ
B_1	$\rightarrow$	$-\frac{1}{2}\Delta$
$B_{00}(k^2, m_0^2, m_1^2)$	$\rightarrow$	$-\frac{1}{4}(k^2/3 - m_0^2 - m_1^2)\Delta$
B_{11}	$\rightarrow$	$\frac{1}{3}\Delta$
C_{00}	$\rightarrow$	$\frac{1}{4}\Delta$

Table 5.1: UV divergent coefficients of the Passarino-Veltman integrals

The IR divergent parts are shown in the Table 5.2. The new parameters presented there are

$$\kappa = \kappa(m_0^2, m_1^2, m_2^2) = \sqrt{\lambda(m_0^2, m_1^2, m_2^2)} \tag{5.6}$$
$$\beta_0 = \frac{m_0^2 - m_1^2 - m_2^2 + \kappa}{2m_1 m_2} \tag{5.7}$$

Integral		IR divergent part
$\dot{B}_0(m^2,\lambda^2,m^2) = \dot{B}_0(m^2,m^2,\lambda^2)$	$\rightarrow$	$-\frac{\ln\lambda^2}{2m^2}$
$\dot{B}_1(m^2,m^2,\lambda^2)$	$\rightarrow$	$\frac{\ln\lambda^2}{2m^2}$
$\dot{B}_1(m^2,\lambda^2,m^2)$	$\rightarrow$	0
$\mathrm{Re}[C_0(m_1^2,m_0^2,m_2^2,\lambda^2,m_1^2,m_2^2)]$	$\rightarrow$	$-\frac{\ln\beta_0}{\kappa}\ln\lambda^2$

Table 5.2: IR divergent coefficients of the Passarino-Veltman integrals

In the Introduction we have mentioned that in supersymmetic models superfields posses equal number of fermionic and bosonic degrees of freedom. But when we work in DREG this no more holds. The reason is that the vector fields become D-dimensional and cannot be combined with its fermionic partner to a superfield. This leads to the need of a new regularization scheme. Such was introduced by the W.Siegel [15] and is called dimensionaal reduction (DRED). In this scheme one still calculates the integrals in D dimensions but the vector fields are kept 4-dimensional. At one loop level when calculating integrals the difference between DREG and DRED can be seen only in the finite terms.

5.2 Renormalization of fermions

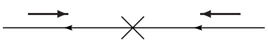
In this section we are going to renormalize wave function of a fermion field as well as mass of a fermion. We use the multiplicative renormalization where the bare parameters are split to the renormalized parameters and their counterterms. This scheme is described in [16]. We will not consider the mixing of the fermions because we will not come across the situation in calculating neutralino decay which will require the mixing. The bare parameters go to the following terms

$$
\begin{aligned}
f_0 &\rightarrow (1+\frac{1}{2}\delta Z^L P_L + \frac{1}{2}\delta Z^R P_R) f && (5.8)\\
\bar{f}_0 &\rightarrow \bar{f}(1+\frac{1}{2}\delta Z^{L\dagger} P_R + \frac{1}{2}\delta Z^{R\dagger} P_L) && (5.9)\\
m_0 &\rightarrow m+\delta m && (5.10)
\end{aligned}
$$

The parameter m is the renormalized mass and as we know that the bare mass m_0 is in fact an infinite parameter it follows that the countertem is as well. The f is the renormalized wave function and is connected to the bare one through the relation $f_0 \rightarrow \sqrt{Z}f$. The original bare Dirac's lagrangian splits to the formally identical lagrangian but with renormalized fields and to the counterterms

$$
\begin{aligned}
\bar{f}_0(i\partial\!\!\!/ - m)f_0 \rightarrow\ & \bar{f}(i\partial\!\!\!/ - m)f \\
+\ & \bar{f}p\!\!\!/(\frac{1}{2}\delta Z^{L\dagger}P_L + \frac{1}{2}\delta Z^{R\dagger}P_R)f - m\bar{f}(\frac{1}{2}\delta Z^{L\dagger}P_R + \frac{1}{2}\delta Z^{R\dagger}P_L)f \\
+\ & \bar{f}(p\!\!\!/ - m)(\frac{1}{2}\delta Z^{L\dagger}P_L + \frac{1}{2}\delta Z^{R\dagger}P_R)f - \delta m\bar{f}f \qquad (5.11)
\end{aligned}
$$

We have obtained the additional Feynmam rule for the counterterm vertex which is conventionally denoted by a cross

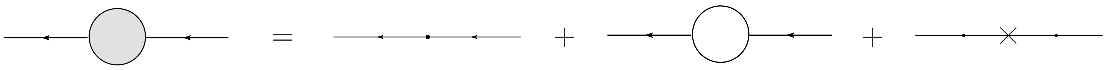


$$i\not{p}\left(\frac{1}{2}\delta Z^{L\dagger}P_L + \frac{1}{2}\delta Z^{R\dagger}P_R\right) + i(\not{p}-m)\left(\frac{1}{2}\delta Z^{L}P_L + \frac{1}{2}\delta Z^{R}P_R\right)$$
$$-im\left(\tfrac{1}{2}\delta Z^{L\dagger}P_R + \tfrac{1}{2}\delta Z^{R\dagger}P_L\right) - i\delta m = i\,\delta\Gamma(p,p)$$

The renormalized Green function is connected with the two point function $\hat{\Gamma}$ through the following relation

$$\hat{G} \;=\; iS(p) + i[S(p)\hat{\Pi}(p^2)S(p)] + \ldots = iS(p)[-i\hat{\Gamma}]iS(p) \tag{5.12}$$

where only one particle irreducible loop diagrams enter into the series. $\hat{\Pi}(p^2)$ is the renormalized self-energy and $iS(p)$ is the fermionic propagator $i(\not{p}-m)^{-1}$. The renormalized amplitude is defined by the following picture



$$i\bar{u}(p)\hat{\Gamma}(-p,p)u(p) \;=\; i\bar{u}(p)\Gamma(-p,p)u(p) \;+\; i\bar{u}(p)\delta\Gamma(-p,p)u(p)$$

$$i\bar{u}(p)\hat{\Gamma}(-p,p)u(p) \;=\; i\bar{u}(p)(\not{p}-m)u(p) \;+\; i\bar{u}(p)\hat{\Pi}(p^2)u(p) \;=\; \mathcal{M}$$

The renormalized self-energies $\hat{\Pi}(p^2)$ can be further decomposed to the following parts

$$\hat{\Pi}(p^2) = \not{p}P_L\hat{\Pi}^L(p^2) + \not{p}P_R\hat{\Pi}^R(p^2) + \hat{\Pi}^{S,L}(p^2)P_L + \hat{\Pi}^{S,R}(p^2)P_R \tag{5.13}$$

They consist of divergent loop diagrams and the corresponding counterterms as follows

$$\hat{\Pi}^{L/R} \;=\; \Pi^{L/R} + \frac{1}{2}\Big(\delta Z^{L/R} + \delta Z^{L/R\dagger}\Big) \tag{5.14}$$

$$\hat{\Pi}^{S,L/R} \;=\; \Pi^{S,L/R} - \frac{1}{2}m\Big(\delta Z^{L/R} + \delta Z^{R/L\dagger}\Big) - \delta m \tag{5.15}$$

To fix the mass-counterterm we use the following on-shell renormalization condition for the physical mass of the fermion (the physical mass of the particle is taken to be the pole of the propagator)

$$\widetilde{\mathrm{Re}}\hat{\Gamma}(p)u(p)\Big|_{p^2=m^2} = 0 \tag{5.16}$$

where $\widetilde{\mathrm{Re}}$ means taking the real part from the loop integrals only. From the previous condition we obtain two following relations

$$P_R: \quad m\Pi^L + \Pi^{S,R} + \frac{1}{2}m(\delta Z^L - \delta Z^R) - \delta m = 0 \tag{5.17}$$

$$P_L: \quad m\Pi^R + \Pi^{S,L} + \frac{1}{2}m(\delta Z^R - \delta Z^L) - \delta m = 0 \tag{5.18}$$

By summing the two relations and dividing the sum by two we come to the final expression for the mass countertem

$$\delta m = \frac{1}{2}\widetilde{\mathrm{Re}}\Big(m\Pi^L(m^2) + m\Pi^R(m^2) + \Pi^{S,L}(m^2) + \Pi^{S,R}(m^2)\Big) \tag{5.19}$$

The renormalization condition for the wave function (the residuum of the propagator by $\not{p} = m$ equals one) reads

$$\lim_{p^2 \to m^2} \frac{1}{\not{p} - m} \widetilde{\mathrm{Re}} \hat{\Gamma}(p) u(p) = u(p) \tag{5.20}$$

Substituting back for the $\hat{\Gamma}$ after some small modifications the left hand side becomes

$$\begin{aligned} P_L: \quad & \Big[\Pi^L(m^2) + \frac{1}{2}\delta Z^L + \frac{1}{2}\delta Z^{L\dagger} + \lim_{p^2 \to m^2} \frac{1}{\not{p} - m}\Big(m\Pi^L(p^2) + m\frac{1}{2}\delta Z^{L\dagger} \\ & \qquad \Pi^{S,L}(p^2) - m\frac{1}{2}\delta Z^{R\dagger} - \delta m\Big)\Big] u(p) \qquad (5.21) \\ P_R: \quad & \Big[\Pi^R(m^2) + \frac{1}{2}\delta Z^R + \frac{1}{2}\delta Z^{R\dagger} + \lim_{p^2 \to m^2} \frac{1}{\not{p} - m}\Big(m\Pi^R(p^2) + m\frac{1}{2}\delta Z^{R\dagger} \\ & \qquad \Pi^{S,R}(p^2) - m\frac{1}{2}\delta Z^{L\dagger} - \delta m\Big)\Big] u(p) \qquad (5.22) \end{aligned}$$

After substituting (5.17) to the first equation and (5.18) to the second, rewriting $(\not{p} - m)^{-1}$ to $\frac{\not{p} + m}{p^2 - m^2}$ and finally using the relation $\frac{\partial f(\not{p})}{\partial p^2} = \frac{\not{p}}{2m^2} \frac{\partial f(\not{p})}{\partial \not{p}}$ we obtain the relations for the wave function counterterm we were looking for

$$\begin{aligned} \delta Z^{L/R} = \widetilde{\mathrm{Re}}\Big\{ & -\Pi^{L/R}(m^2) + \frac{1}{2m}\Big(\Pi^{S,L/R}(m^2) - \Pi^{S,R/L}(m^2)\Big) \\ & - \frac{\partial}{\partial p^2}\Big[m^2\Big(\Pi^{L/R}(p^2) + \Pi^{R/L}(p^2)\Big) + m\Big(\Pi^{S,L/R}(p^2) + \Pi^{S,R/L}(p^2)\Big)\Big]\Big|_{p^2 = m^2}\Big\} \end{aligned} \tag{5.23}$$

5.3 Renormalization of scalars

The renormalization of scalars proceeds in an analogous way as in the case of fermions. But this time we will consider the mixing of the scalars too as we will need it later in our calculation of the decay. The bare parameters (on the left) consist of the following terms

$$\tilde{f}_i \to (\delta_{ij} + \frac{1}{2}\delta Z_{ij})\tilde{f}_j \tag{5.24}$$

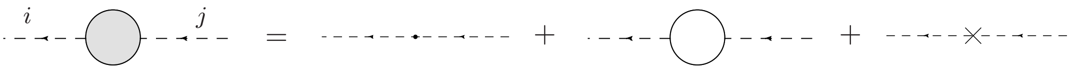
$$\tilde{f}_j^* \to \tilde{f}_k^*(\delta_{kj} + \frac{1}{2}\delta Z_{jk}^*) \tag{5.25}$$

$$m_i^2 \to m_i^2 + \delta m_i^2 \tag{5.26}$$

The original bare lagrangian consists of the two parts - lagrangian containing renormalized fields and their masses and the lagrangian containing counterterms

$$\begin{aligned} \partial_\mu \tilde{f}_i^* \partial^\mu \tilde{f}_i - m_i^2 \tilde{f}_i^* \tilde{f}_i \quad \to \quad & \partial_\mu \tilde{f}_i^* \partial^\mu \tilde{f}_i - m_i^2 \tilde{f}_i^* \tilde{f}_i + \frac{1}{2}(\delta Z_{ij} + \delta Z_{ji}^*)(\partial_\mu \tilde{f}_i^*)(\partial^\mu \tilde{f}_j) \\ - \quad & \frac{1}{2}(m_i^2 \delta Z_{ij} + m_j^2 \delta Z_{ji}^*)\tilde{f}_i^* \tilde{f}_j - \delta m_i^2 \delta_{ij} \tilde{f}_i^* \tilde{f}_j \end{aligned} \tag{5.27}$$

The renormalized amplitude in the case of sfermions is defined in the following picture



$$i\hat{\Gamma}_{ij}(p,-p) \quad = \quad i(p^2-m_i^2)\delta_{ij} \quad + \quad i\hat{\Pi}_{ij} \quad = \quad \mathcal{M}$$

The renormalized self-energy consists of divergent loop diagrams and counterterms

$$\hat{\Pi}_{ij} = \Pi_{ij} + \frac{1}{2}(p^2 - m_i^2)\delta Z_{ij} + \frac{1}{2}(p^2 - m_j^2)\delta Z^*_{ji} - \delta_{ij}\delta m_i^2 \tag{5.28}$$

The on-shell renormalization conditions for the scalars are as follows

$$\widetilde{\mathrm{Re}}\hat{\Gamma}_{ij}(p^2)\Big|_{p^2=m_j^2} = 0 \qquad\qquad \lim_{k^2\to m_i^2} \frac{\widetilde{\mathrm{Re}}\hat{\Gamma}_{ii}(p^2)}{k^2 - m_i^2} = 1 \tag{5.29}$$

These conditions restrict the counterterms to be

$$\delta m_i^2 \;=\; \widetilde{\mathrm{Re}}\Pi_{ii}(m_i^2) \tag{5.30}$$

$$\delta Z_{ij} \;=\; \frac{2}{m_i^2 - m_j^2}\widetilde{\mathrm{Re}}\,\Pi_{ij}(m_j^2) \qquad i \neq j \tag{5.31}$$

$$\delta Z_{ii} \;=\; -\widetilde{\mathrm{Re}}\,\frac{\partial}{\partial p^2}\Pi_{ii}(p^2)\Big|_{p^2=m_i^2} \tag{5.32}$$

The wave function and mass of the scalars is not everything what has to be renormalized. There is another parameter one cannot forget - mixing angle $\theta_{\tilde{f}}$. The counterterm to the mixing matrix is set to cancel the anti-hermitian part of the wave function correction

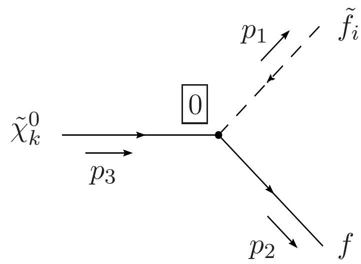
$$\delta R^{\tilde{f}}_{ij} = \sum_{k=1}^{2} \frac{1}{4}(\delta Z_{ik} - \delta Z_{ki})R^{\tilde{f}}_{kj} \tag{5.33}$$

Chapter 6

The neutralino decay

6.1 Tree level

The Feynnman diagram for the neutralino decay to a fermion and a sfermion is



$$\boxed{0}: \; -i\Big(a_{ik}^{\tilde{f}} P_R + b_{ik}^{\tilde{f}} P_L\Big)$$

In this chapter we fix the indices i, k which designate neutralino and sfermion participated in the process. The amplitude for this decay within a tree level without considering color of particles is

$$\mathcal{M}_0 = -i\,\bar{u}(p_2)\Big(a_{ik}^{\tilde{f}} P_R + b_{ik}^{\tilde{f}} P_L\Big)u(p_3) \tag{6.1}$$

We will count the decay width for unpolarized case therefore we average through neutralino spin states and sum over fermion spin states

$$\begin{aligned} |\overline{\mathcal{M}_0}|^2 &= \frac{1}{2}\sum_{s_3,s_2} \mathcal{M}_0 \mathcal{M}_0^* \\ &= \frac{1}{2}\mathrm{Tr}\Big[(\not{p}_2 + m_f)(a_{ik}^{\tilde{f}} P_R + b_{ik}^{\tilde{f}} P_L)(\not{p}_3 + m_{\tilde{\chi}_k^0})(a_{ik}^{*\tilde{f}} P_L + b_{ik}^{*\tilde{f}} P_R)\Big] \\ &= p_2 \cdot p_3\,(|a_{ik}^{\tilde{f}}|^2 + |b_{ik}^{\tilde{f}}|^2) + m_f m_{\tilde{\chi}_k^0}\,(a_{ik}^{\tilde{f}} b_{ik}^{*\tilde{f}} + a_{ik}^{*\tilde{f}} b_{ik}^{\tilde{f}}) \end{aligned} \tag{6.2}$$

We stress again that we do not sum over the indices i, k. Finally we come to the tree level decay width in the CMS system

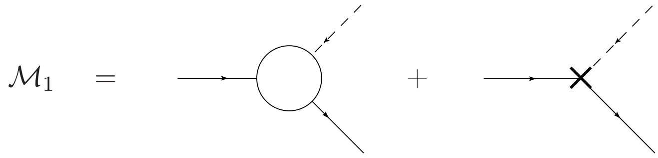
$$\Gamma_0 = \frac{p_f}{32\pi^2 m_{\tilde{\chi}_k^0}^2} \int |\overline{\mathcal{M}_0}|^2 d\Omega \tag{6.3}$$

where the resulting particles carry the momentum of the absolute value p_f

$$\begin{aligned} p_f &= \frac{\sqrt{\lambda(m^2_{\tilde{\chi}^0_k}, m^2_{\tilde{f}_i}, m^2_f)}}{2m_{\tilde{\chi}^0_k}} && (6.4) \\ \lambda(x,y,z) &= x^2 + y^2 + z^2 - 2xy - 2yz - 2zx && (6.5) \end{aligned}$$

6.2 One loop level

To calculate the amplitude at one loop level we must treat the following two diagrams



The first diagram leads to vertex corrections, the second diagram includes wave function and other counterterms. The external fields and their momenta are the same as in the tree level case. We define new renormalized coefficients A, B and write the one loop level amplitude in the following manner

$$\mathcal{M}_1 = -i\,\bar{u}(p_2)(AP_R + BP_L)u(p_3) \tag{6.6}$$

The coefficients A, B further decompose to three parts: vertex corrections, wave function corrections and conterterm corrections

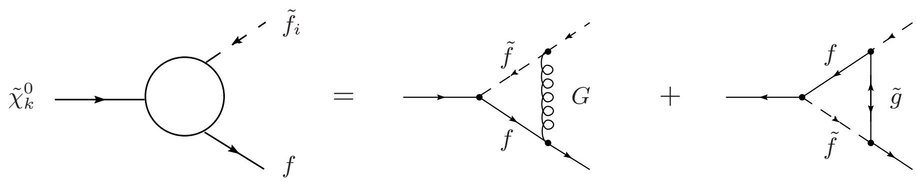
$$\begin{aligned} A &= a^{(v)} + a^{(w)} + a^{(c)} && (6.7) \\ B &= b^{(v)} + b^{(w)} + b^{(c)} && (6.8) \end{aligned}$$

The absolute value of the amplitude to the next to tree level is

$$\begin{aligned} |\mathcal{M}|^2 &= |\mathcal{M}_0|^2 + 2\,\mathrm{Re}[\mathcal{M}_0^*\mathcal{M}_1] + \ldots \\ &= p_2.p_3\,(|a^{\tilde{f}}_{ik}|^2 + |b^{\tilde{f}}_{ik}|^2) + m_f m_{\tilde{\chi}^0_k}\,(a^{\tilde{f}}_{ik} b^{\tilde{f}*}_{ik} + a^{\tilde{f}*}_{ik} b^{\tilde{f}}_{ik}) \\ &+ 2\,p_2.p_3\,\mathrm{Re}[Aa^{\tilde{f}*}_{ik} + Bb^{\tilde{f}*}_{ik}] + 2\,m_f m_{\tilde{\chi}^0_k}\,\mathrm{Re}[Ab^{\tilde{f}*}_{ik} + a^{\tilde{f}*}_{ik}B] + \ldots && (6.9) \end{aligned}$$

Vertex corrections

There are two diagrams that contribute to the vertex corrections



The coefficients $a^{(v)}, b^{(v)}$ are (see the appendix B)

$$a^{(v)} = (4\pi)^2 A_R^{f\tilde{f}G}(\lambda, m_{\tilde{f}_i}, m_f, b^{\tilde{f}}_{ik}, a^{\tilde{f}}_{ik}, -g_s, -g_s, -g_s)$$

$$
\begin{aligned}
&+ \quad (4\pi)^2 \sum_{j=1}^{2} A_R^{\tilde g f \tilde f}(m_{\tilde g}, m_f, m_{\tilde f_j}, a^{\tilde f}_{jk}, b^{\tilde f}_{jk}, \sqrt{2} g_s R^{\tilde q}_{iR}, -\sqrt{2} g_s R^{\tilde q}_{iL}, \\
&\qquad \sqrt{2} g_s R^{\tilde q}_{jR}, -\sqrt{2} g_s R^{\tilde q}_{jL}) \qquad (6.10)
\end{aligned}
$$

$$
b^{(v)} = a^{(v)}(A_R^{f\tilde f G} \leftrightarrow A_L^{f\tilde f G},\ A_R^{\tilde g f \tilde f} \leftrightarrow A_L^{\tilde g f \tilde f}) \qquad (6.11)
$$

The second diagram needed a special treatment. The neutralino is a Majorana fermion and not only the contractions $\bar{\tilde g}\tilde g$, $\tilde g\bar{\tilde g}$ are allowed but also the contractions $\tilde g\tilde g$ and $\bar{\tilde g}\bar{\tilde g}$ are possible. The rules for Majorana fermions are described in [17]. According to this text it is more convenient to transform terms in the corresponding T-product. The second diagram for vertex correction can be transformed in the following way

$$
\ldots \bar f \Gamma_1 \tilde g \tilde f \,|\, \bar f \Gamma_2 \tilde g \tilde f \,|\, \bar{\tilde\chi} \Gamma_3 f \tilde f^* \ldots \to \ldots \bar f \Gamma_1 \tilde g \tilde f \,|\, \bar{\tilde g}^c \Gamma_2' f^c \tilde f \,|\, \bar f^c \Gamma_3' \tilde\chi^c \tilde f^* \ldots
$$

where $\Gamma_i' = C\Gamma_i C^{-1} = \eta_i \Gamma_i\,, C$ is the charge conjugation operator and $\eta_i = 1$ for $1, \gamma_5$.

The Dirac spinor and the charge-conjugate Dirac spinor are

$$
\psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_p^s u^s(p) e^{-ipx} + b_p^{\dagger s} v^s(p) e^{ipx}) \qquad (6.12)
$$

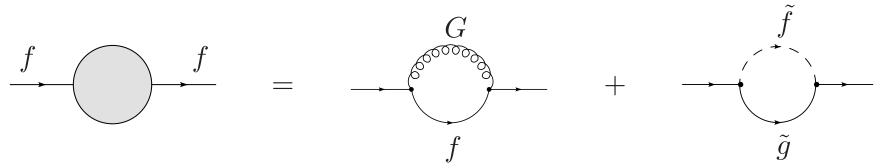
$$
\psi^c(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_p^{\dagger s} v^s(p) e^{ipx} + b_p^s u^s(p) e^{-ipx}) \qquad (6.13)
$$

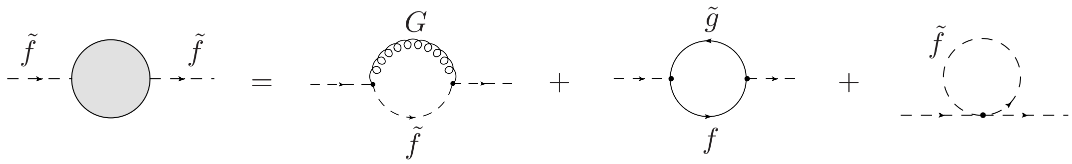
And the expression for the propagator of the charge-conjugate fermion field is

$$
\langle 0|T(f^c \bar f^c)|0\rangle = C(\langle 0|T(f\bar f)|0\rangle)^T C^{-1} = \frac{i}{-\not p - m} = iS(-p) \qquad (6.14)
$$

Wave function corrections

Two diagrams contribute to the fermion self-energy and three to the sfermion





We have not explicitly showed the indices on sfermions that denote mass eigenstate.

The coefficients $a^{(w)}, b^{(w)}$ are (see the appendix B)

$$
a^{(w)} = \frac{1}{2}(\delta Z^L_{fG} + \sum_{n=1}^{2} \delta Z^L_{\tilde g \tilde f_n})\, a^{\tilde f}_{ik} + \frac{1}{2}\sum_{j=1}^{2}(\delta Z^{\tilde f_i G}_{ji} + \delta Z^{\tilde g f}_{ji} + \sum_{n=1}^{2} \delta Z^{\tilde f_n}_{ji}) a^{\tilde f}_{jk} \qquad (6.15)
$$

$$
b^{(w)} = a^{(w)}(L \leftrightarrow R, a^{\tilde f}_{ik} \leftrightarrow b^{\tilde f}_{ik}, a^{\tilde f}_{jk} \leftrightarrow b^{\tilde f}_{jk}) \qquad (6.16)
$$

where the arguments that appear in the functions $\Pi(p^2, \ldots)$ are

$$\begin{array}{lcc}
\delta Z_{fG} & \leftrightarrow & \Pi(m_f^2, \lambda, m_f, -g_s, -g_s, -g_s, -g_s) \\
\delta Z_{\tilde{g}\tilde{f}_n} & \leftrightarrow & \Pi(m_f^2, m_{\tilde{f}_n}, m_{\tilde{g}}, -\sqrt{2}g_s R^{\tilde{q}}_{nL}, \sqrt{2}g_s R^{\tilde{q}}_{nR}, \sqrt{2}g_s R^{\tilde{q}}_{nR}, -\sqrt{2}g_s R^{\tilde{q}}_{nL}) \\
\delta Z^{fG}_{ji} & \leftrightarrow & \Pi(m^2_{\tilde{f}_i}, \lambda, m_{\tilde{f}_i}, -g_s, -g_s) \\
\delta Z^{\tilde{g}f}_{ji} & \leftrightarrow & \Pi(m^2_{\tilde{f}_i}, m_{\tilde{g}}, m_f, \sqrt{2}g_s R^{\tilde{q}}_{iR}, -\sqrt{2}g_s R^{\tilde{q}}_{iL}, -\sqrt{2}g_s R^{\tilde{q}}_{jL}, \sqrt{2}g_s R^{\tilde{q}}_{jR}) \\
\delta Z^{\tilde{f}_n}_{ji} & \leftrightarrow & \Pi(\ldots, m_{\tilde{f}_n}, -g_s^2 A^2_{jn} A^2_{ni})
\end{array}$$

In the topology with sfermion scalar loop we have omitted the possibility of presence of the other flavor of squark in the loop. This is prohibited because the color factor equals zero. We also do not need to consider the term $A^2_{ji}A^2_{nn}$ in the four sfermions coupling from the same reason.

Counterterm corrections

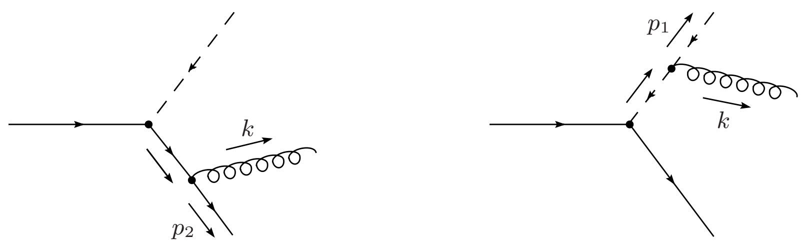
$$a^{(c)} = \frac{1}{m_f} h_f \delta m_f Z_{k3} R^{\tilde{f}}_{i2} + h_f Z_{k3} \delta R^{\tilde{f}}_{i2} + g f^f_{Lk} \delta R^{\tilde{f}}_{i1} \tag{6.17}$$

$$b^{(c)} = \frac{1}{m_f} h_f \delta m_f Z_{k3} R^{\tilde{f}}_{i1} + h_f Z_{k3} \delta R^{\tilde{f}}_{i1} + g f^f_{Rk} \delta R^{\tilde{f}}_{i2} \tag{6.18}$$

6.3 Soft gluon radiation

Having calculated the decay width at one loop level including all the wave function and counterterm corrections the result is free of UV-divergence. But there is another type of the divergence - infrared one. This is caused by the appearance of the massless particle (in our case it is gluon) in the loops. This divergence is compensated when we calculate a sum of two decay widths the original one and a one with an additional gluon in the final state carrying infinitely small energy and thus being undetectable. Such gluons are called soft.

Gluon can be radiated by fermion as well as by the sfermion



The amplitude for the first process is

$$\begin{aligned}
\mathcal{M}_f &= \bar{u}(p_2)(-ig_s\gamma^\mu)\,\varepsilon^*_\mu \frac{i(\not{p}_2 + \not{k} + m_f)}{(p_2+k)^2 - m_f^2} \mathcal{A}_0(p_2+k) \\
&= \bar{u}(p_2)(-ig_s\gamma^\mu)\,\varepsilon^*_\mu \frac{i(\not{p}_2 + m_f)}{2p_2.k} \mathcal{A}_0(p_2) \;= \bar{u}(p_2)\; g_s \frac{p_2.\varepsilon^*}{p_2.k} \mathcal{A}_0(p_2)
\end{aligned} \tag{6.19}$$

The $\mathcal{A}_0$ is connected to the process without an additional gluon by the relation

$$\mathcal{M}_0 = \bar{u}(p_2)\,\mathcal{A}_0 \tag{6.20}$$

Analogously the amplitude in the case of radiated gluon by sfermion is

$$\mathcal{M}_{\tilde{f}} = -g_s \frac{p_1.\varepsilon^*}{p_1.k}\,\mathcal{A'}_0(p_1) \tag{6.21}$$

In both amplitudes we have not included the generator T^a. We will count the color factor in the next section.

The square of the amplitude of radiating gluon in unpolarized case is

$$\overline{|\mathcal{M}_{\rm soft}|^2} = \overline{|\mathcal{M}_0|^2}\,(-g_s^2)\left\{\frac{p_2^2}{(p_2.k)^2} - \frac{2p_2.p_1}{(p_2.k)(p_1.k)} + \frac{p_1^2}{(p_1.k)^2}\right\} \tag{6.22}$$

where the minus sign in $(-g_s^2)$ comes from using the formula

$$\sum_\lambda \varepsilon_\mu^{*\lambda}(k)\varepsilon_\nu^\lambda(k) = -g_{\mu\nu} \tag{6.23}$$

The result for the soft gluon radiation can be written in the following form [18]

$$\left(\frac{d\Gamma}{d\Omega}\right)_{\rm soft} = -\left(\frac{d\Gamma}{d\Omega}\right)_0 \times \frac{g_s^2}{(2\pi)^3}\int_{|\vec{k}|\le\Delta E}\frac{d^3k}{2\omega}\,T \tag{6.24}$$

where

$$T = \left\{\frac{p_2^2}{(p_2.k)^2} - \frac{2p_2.p_1}{(p_2.k)(p_1.k)} + \frac{p_1^2}{(p_1.k)^2}\right\} \tag{6.25}$$

The integrals needed for the calculation have generally the form

$$I_{ab} = \int_{|\vec{k}|\le\Delta E}\frac{d^3k}{2\omega}\,\frac{2a.b}{a.k\,b.k} \tag{6.26}$$

When $a = b$ the integral equals

$$I_{a^2} = 2\pi\left\{\log\frac{4\Delta E^2}{\lambda^2} + \frac{a_0}{|\vec{a}|}\log\frac{a_0-|\vec{a}|}{a_0+|\vec{a}|}\right\} \tag{6.27}$$

This integral is divergent after sending the gluon mass λ to zero. However, this IR-divergence cancel with the IR-divergence in the B-integrals presented in the self-energy diagrams.

The second special case is when $\vec{a} = -\vec{b} = \vec{p}$

$$\begin{aligned} I_{ab} &= 2\pi\frac{a.b}{(a_0+b_0)|\vec{p}|}\left\{\frac{1}{2}\log\frac{a_0+|\vec{p}|}{a_0-|\vec{p}|}\log\frac{4\Delta E^2}{\lambda^2} - \mathrm{Li}_2\left(\frac{2|\vec{p}|}{a_0+|\vec{p}|}\right) - \frac{1}{4}\log^2\frac{a_0+|\vec{p}|}{a_0-|\vec{p}|}\right. \\ &\left. + \frac{1}{2}\log\frac{a_0+|\vec{p}|}{a_0-|\vec{p}|}\log\frac{4\Delta E^2}{\lambda^2} - \mathrm{Li}_2\left(\frac{2|\vec{p}|}{a_0+|\vec{p}|}\right) - \frac{1}{4}\log^2\frac{a_0+|\vec{p}|}{a_0-|\vec{p}|}\right\} \end{aligned} \tag{6.28}$$

Divergent parts from this integral cancel with divergent parts in C-integrals in vertex corrections.

It is standard to write the equation (6.24) as

$$\left(\frac{d\Gamma}{d\Omega}\right)_{\text{soft}} = \left(\frac{d\Gamma}{d\Omega}\right)_0 \delta_s \tag{6.29}$$

where δ_s is defined as

$$\delta_s = \frac{-g_s^2}{(2\pi)^3\,2}(I_{p_2^2} - 2I_{p_2p_1} + I_{p_1^2}) \tag{6.30}$$

6.4 Neutralino decay width

The neutralino decay width with corrections to the second power of the coupling g_s is

$$\Gamma = \frac{4\pi p_f}{32\pi^2 m_{\tilde{\chi}_k^0}^2}\left(C_F^0|\overline{\mathcal{M}_0}|^2 + C_F^s\delta_s|\overline{\mathcal{M}_0}|^2 + C_F^1 2\mathrm{Re}[\mathcal{M}_0^*\mathcal{M}_1]\right) \tag{6.31}$$

We have kept postponing the identification of the color factors through the previous section. However, it is always useful to think of them at the beginning as some could equal zero which can simplify the calculation.

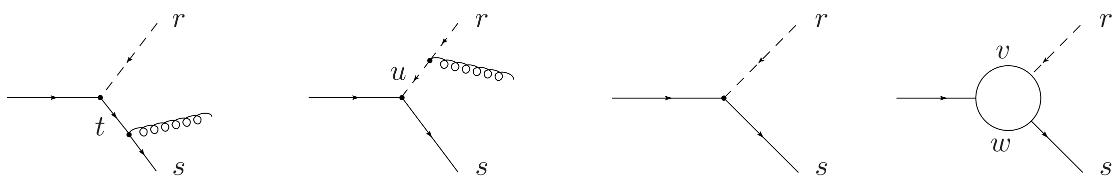
The color factor for tree level case is the simplest one. It only declares the fact that there are three possible color states of the final particles

$$C_F^0 = 3 \tag{6.32}$$

The helpful relation for counting rest color factors is the identity

$$\sum_{a,t} T_{st}^a T_{tu}^a = \frac{4}{3}\delta_{su} \tag{6.33}$$

The following diagrams include the color of particles. They represent amplitudes in this order: $\mathcal{M}_f$, $\mathcal{M}_{\tilde{f}}$, $\mathcal{M}_0$, $\mathcal{M}_1$



The second color factor C_F^s is the same for $|\mathcal{M}_f|^2$, $|\mathcal{M}_{\tilde{f}}|^2$ and $\mathcal{M}_f\mathcal{M}_{\tilde{f}}^*$

$$\mathcal{M}_f\mathcal{M}_{\tilde{f}}^* : \qquad C_F^s = \sum_{a,r,s,t,u} \delta_{tr}\delta_{su} T_{st}^a T_{ur}^{*a} = \sum_{a,r,s} T_{sr}^a T_{rs}^a = 4 \tag{6.34}$$

The last color factor C_F^1 is

$$C_F^1 = \sum_{a,r,s,v,w} \delta_{rs}\delta_{vw} T_{vr}^a T_{sw}^a = 4 \tag{6.35}$$

6.5 Numerical results

In this subsection I present the numerical results for the neutralino decay. There are lots of new parameters entering the MSSM theory. I set their numerical values at first. I have chosen for the values given in the MSSM package of the Feynarts program for the most of them.

The first set of the presented graphs shows the decay widths and masses of the particles as a function of the parameter μ. This parameter is involved in many places: Higgs potential, neutralino as well as sfermion masses, etc. The parameter $\tan\beta$ is set to be 7.

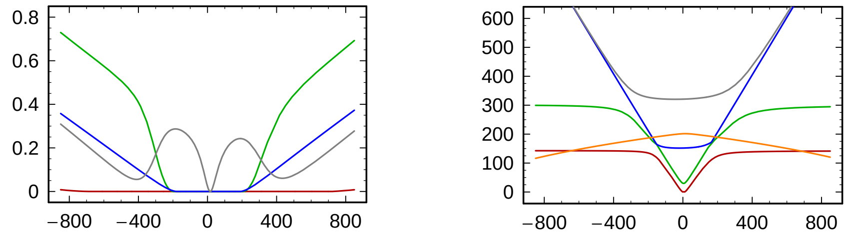


Figure 6.1: **left graph:** Γ_0 (GeV) as a function of the parameter μ; **right graph:** mass (absolute value) spectrum; each **color** represent the following particle: red - neutralino 1, green - neutralino 2, blue - neutralino 3, gray - neutralino 4, orange - sbottom 1

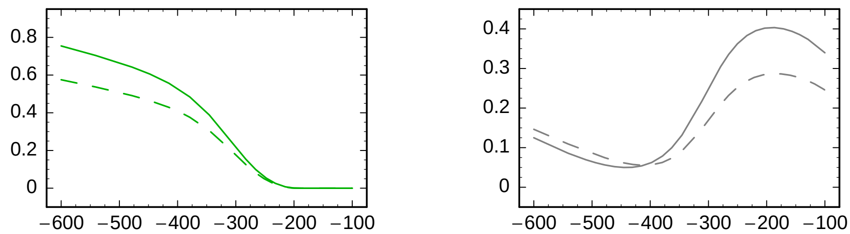


Figure 6.2: **left graph:** Γ_0 (dashed) and Γ (solid) for the neutralino 2; **right graph:** Γ_0 (dashed) and Γ (solid) for neutralino 4 as a function of the parameter μ

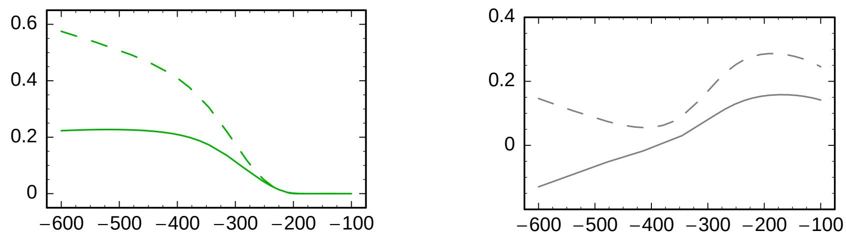


Figure 6.3: The situation as in the Fig.6.2 but with omitting the finite terms coming from gluon radiation

In the four upper graphs I let the parameter $\tan\beta$ to vary. The parameter μ is set to be -400.

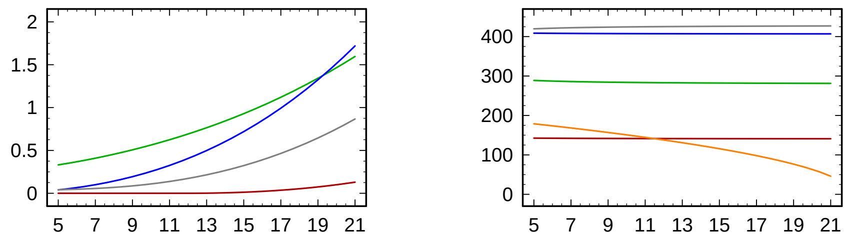


Figure 6.4: **left graph:** Γ_0 (GeV) as a function of the parameter $\tan\beta$; **right graph:** mass (absolute value) spectrum (GeV)

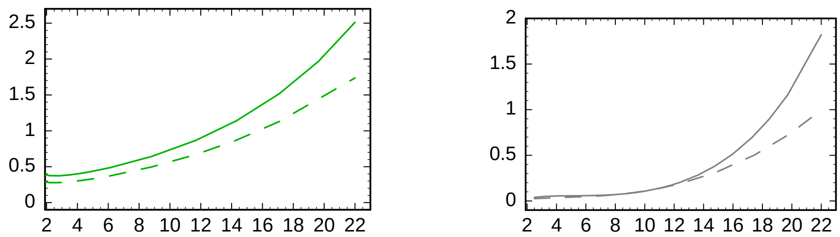


Figure 6.5: **left graph:** Γ_0 (dashed) and Γ (solid) for the neutralino 2; **right graph:** Γ_0 (dashed) and Γ (solid) for the neutralino 4 as a function of the parameter $\tan\beta$

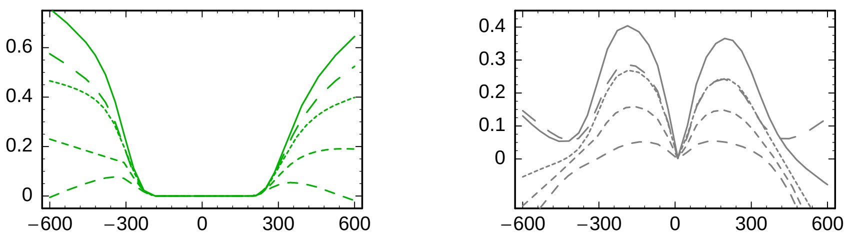


Figure 6.6: **left graph:** Γ_0 (longest dashes), Γ with included bremsstrahlung (solid) and Γs with included soft gluon approximation with different values of $\Delta E^2 (\Delta E^2 = 1, 10, 100\text{GeV}$ from the bottom curve up) for the neutralino 2 and 4; $\tan\beta = 7$

parameter point: $M = 300, M_{Susy} = 200, A_e = A_u = A_d = 100, M_{\tilde{g}} = 787, m_{A_0} = 700, m_{H^+} = 705$

From the presented graphs we see that the corrections to the tree level widths are around 30%. From the last two graphs we can see that the soft gluon approximation is better with increasing gluon energy. But gluon with the energy around 10 GeV can no more be considered soft.

Conclusion

In my master thesis I derived the lagrangian for the MSSM theory using the formalism of superspace and superfields. I tried to be as close to the notation that is used at the Institute for High Energy Physics in Vienna as possible.

I calculated the decay width for the neutralino decay to an antisbottom and a bottom quark at a one loop level considering only the QCD corrections using Feynman rules coming from the derived lagrangian. I had to consider two loop diagrams contributing to the vertex corrections and five loop diagrams contributing to the self-energies those generic form I calculated by hand. I also encountered the problem with an infrared divergence which I solved by considering a soft gluon radiation.

At the end of my thesis I presented graphs where the decay widths and particle masses depend on various MSSM parameters.

Appendix A

Notation, Spinor Algebra and Grassman numbers

We use the metric $\eta_{\mu\nu} = diag(1, -1, -1, -1)$.

The Pauli matrices and the matrix σ^0 are defined as

$$\sigma^0 := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \sigma^1 := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

In the Weyl representation, the Dirac matrices γ^μ are given by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

where $\sigma^\mu := (\sigma^0, \sigma^i)$, and $\bar{\sigma}^\mu := (\sigma^0, -\sigma^i)$

Dirac bispinor: $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$

The two-component objects ψ_L and ψ_R are called left-handed and right-handed Weyl spinors. Their transformation laws under rotations $\vec{\alpha}$ and boosts $\vec{\beta}$ are

$$\begin{array}{llrcl} \psi_L \to A\psi_L & \text{where} & A & = & \exp(-\frac{i}{2}\vec{\alpha}\vec{\sigma} - \frac{1}{2}\vec{\beta}\vec{\sigma}) \\ \psi_R \to (A^{-1})^+\psi_R & \text{where} & (A^+)^{-1} & = & \exp(-\frac{i}{2}\vec{\alpha}\vec{\sigma} + \frac{1}{2}\vec{\beta}\vec{\sigma}) \end{array}$$

There are two inequivalent spinor representations of SL(2,C), the self-representation and the complex conjugate self-representation. Elements of the representation space transform under the self-representation as

$$\chi_a \to A_a{}^b \chi_b$$

and under the complex conjugate self-representation as

$$\eta_{\dot{a}} \to A^*{}_{\dot{a}}{}^{\dot{b}} \eta_{\dot{b}} = \eta_{\dot{b}} A^*{}_{\dot{a}}{}^{\dot{b}} = \eta_{\dot{b}} A^{*T\dot{b}}{}_{\dot{a}} = \eta_{\dot{b}} A^{+\dot{b}}{}_{\dot{a}}$$

Our spinor summation convention is: $\chi\eta = \chi^a \eta_a = \eta\chi$

$$\bar{\chi}\bar{\eta} = \chi_{\dot{a}}\eta^{\dot{a}} = \bar{\eta}\bar{\chi}$$

The components of spinors $\chi, \bar{\eta}, \ldots$ are Grassman variables therefore the quadratic forms $(\chi\eta), (\bar{\chi}\bar{\eta})$ are symmetric.

We rise and lower the spinor indices by using the invariant two-dimensional antisymmetric metric tensor: $\chi^a = \varepsilon^{ab}\chi_b, \chi_a = \varepsilon_{ab}\chi^b, \eta^{\dot{a}} = \varepsilon^{\dot{a}\dot{b}}\eta_{\dot{b}}, \eta_{\dot{a}} = \varepsilon_{\dot{a}\dot{b}}\eta^{\dot{b}}$

$$\varepsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \varepsilon^{\dot{a}\dot{b}} = i\sigma^2 \qquad \varepsilon_{ab} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \varepsilon_{\dot{a}\dot{b}}$$

Let us compute the transformation of $\eta^{\dot{a}}$. We will need the following relation: $(\sigma^i)^T = (\sigma^i)^* = -\sigma^2\sigma^i\sigma^2$

$$\begin{aligned} \eta'^{\dot{a}} = \varepsilon^{\dot{a}\dot{b}}\eta'_{\dot{b}} &= \varepsilon^{\dot{a}\dot{b}}\eta_{\dot{c}}A^{+\dot{c}}{}_{\dot{b}} = \varepsilon^{\dot{a}\dot{b}}\varepsilon_{\dot{c}\dot{d}}\eta^{\dot{d}}A^{+\dot{c}}{}_{\dot{b}} = \varepsilon_{\dot{d}\dot{c}}A^{+\dot{c}}{}_{\dot{b}}\varepsilon^{\dot{b}\dot{a}}\eta^{\dot{d}} \\ &= -i\sigma^2(1 + \tfrac{i}{2}\vec{\alpha}\vec{\sigma} + \tfrac{1}{2}\vec{\beta}\vec{\sigma} + \cdots)i\sigma^2 \\ &= (1 - \tfrac{i}{2}\vec{\alpha}\vec{\sigma}^T + \tfrac{1}{2}\vec{\beta}\vec{\sigma}^T + \cdots)_{\dot{d}}{}^{\dot{a}}\eta^{\dot{d}} \\ &= (1 - \tfrac{i}{2}\vec{\alpha}\vec{\sigma} + \tfrac{1}{2}\vec{\beta}\vec{\sigma} + \cdots)^{\dot{a}}{}_{\dot{d}}\eta^{\dot{d}} = (A^{-1})^{+\dot{a}}{}_{\dot{d}}\eta^{\dot{d}} \end{aligned}$$

Thus we see the index structure in the Dirac bispinor: $(\psi_L)_a, (\psi_R)^{\dot{a}}$

We know that Lorentz group and 2-dimensional special linear group are closely connected. To be more precise, $SL(2,C)$ is the universal covering group of $L_+^\uparrow$ group. For the details we refer to [19]. From the following relation

$$\sigma^\mu \mapsto A\sigma^\mu A^+ = (\Lambda^{-1})^\mu{}_\nu\sigma^\nu, \qquad (\Lambda^{-1})^\mu{}_\nu = \frac{1}{2}Tr(\bar{\sigma}_\nu A\sigma^\mu A^+)$$

we can uncover the index structure of σ matrix:

$$A_a{}^b(\sigma^\mu)_{b\dot{c}}A^{+\dot{c}}{}_{\dot{d}} = ((\Lambda^{-1})^\mu{}_\nu)_a{}^b(\sigma^\nu)_{b\dot{d}}$$

The following relations hold: $$\begin{aligned} (\bar{\sigma}^\mu)^{\dot{a}a} &= \varepsilon^{ab}\varepsilon^{\dot{a}\dot{b}}\sigma^\mu{}_{b\dot{b}} \\ \sigma^\mu{}_{a\dot{a}} &= \varepsilon_{ab}\varepsilon_{\dot{a}\dot{b}}(\bar{\sigma}^\mu)^{\dot{b}b} \end{aligned}$$

The $\sigma^{\mu\nu}, \bar{\sigma}^{\mu\nu}$ are defined as: $$\begin{aligned} \sigma^{\mu\nu} &:= \tfrac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu) \\ \bar{\sigma}^{\mu\nu} &:= \tfrac{i}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu) \end{aligned}$$

Here we present useful identities

$$\begin{aligned} \theta^a\theta^b &= -\frac{1}{2}\varepsilon^{ab}(\theta\theta) & \theta_a\theta_b &= \frac{1}{2}\varepsilon_{ab}(\theta\theta) \\ \theta^{\dot{a}}\theta^{\dot{b}} &= \frac{1}{2}\varepsilon^{\dot{a}\dot{b}}(\bar{\theta}\bar{\theta}) & \theta_{\dot{a}}\theta_{\dot{b}} &= -\frac{1}{2}\varepsilon_{\dot{a}\dot{b}}(\bar{\theta}\bar{\theta}) \end{aligned} \tag{A.1}$$

$$(\sigma^\mu)_{b\dot{a}} = (\bar{\sigma}^\mu)_{\dot{a}b} \tag{A.2}$$
$$Tr(\sigma^\mu\bar{\sigma}^\nu) = 2\eta^{\mu\nu} \tag{A.3}$$
$$\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu = 2\eta^{\mu\nu} \tag{A.4}$$

$$\begin{aligned}
\varepsilon_{ac}(\sigma^{\mu\nu})_b{}^c &= \varepsilon_{bc}(\sigma^{\mu\nu})_a{}^c \\
\varepsilon_{\dot a\dot b}(\bar\sigma^{\mu\nu})^{\dot b}{}_{\dot c} &= \varepsilon_{\dot c\dot b}(\bar\sigma^{\mu\nu})^{\dot b}{}_{\dot a} \qquad (A.5)\\
\theta\sigma^\mu\bar\xi &= -\bar\xi\bar\sigma^\mu\theta \qquad (A.6)\\
(\theta\sigma^\mu\bar\xi)^+ &= (\xi\sigma^\mu\bar\theta) \qquad (A.7)\\
(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) &= \frac{1}{2}\eta^{\mu\nu}(\theta\theta)(\bar\theta\bar\theta) \qquad (A.8)\\
Tr(\bar\sigma^{\mu\nu}\bar\sigma^{\rho\sigma}) &= \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma}-\eta^{\mu\sigma}\eta^{\nu\rho})-\frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}\\
Tr(\sigma^{\mu\nu}\sigma^{\rho\sigma}) &= \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma}-\eta^{\mu\sigma}\eta^{\nu\rho})+\frac{i}{2}\varepsilon^{\mu\nu\rho\sigma} \qquad (A.9)
\end{aligned}$$

In the course of chapter 2 and 3 when constructing lagrangian we will need the following relations

$$\begin{aligned}
\sigma^\mu{}_{a\dot b}\bar\sigma^{\nu\,\dot b c} &= \frac{1}{2}(\sigma^\mu\bar\sigma^\nu+\sigma^\nu\bar\sigma^\mu)_a{}^c+\frac{1}{2}(\sigma^\mu\bar\sigma^\nu-\sigma^\nu\bar\sigma^\mu)_a{}^c = \eta^{\mu\nu}\delta_a{}^c+\frac{2}{i}\sigma^{\mu\nu}{}_a{}^c \qquad (A.10)\\
\bar\sigma^{\mu\,\dot a b}\sigma^\nu{}_{b\dot c} &= \frac{1}{2}(\bar\sigma^\mu\sigma^\nu+\bar\sigma^\nu\sigma^\mu)^{\dot a}{}_{\dot c}+\frac{1}{2}(\bar\sigma^\mu\sigma^\nu-\bar\sigma^\nu\sigma^\mu)^{\dot a}{}_{\dot c} = \eta^{\mu\nu}\delta^{\dot a}{}_{\dot c}+\frac{2}{i}\bar\sigma^{\mu\nu}{}^{\dot a}{}_{\dot c} \qquad (A.11)
\end{aligned}$$

Here we show an useful example of manipulating with Weyl spinors

$$\begin{aligned}
\theta\partial_\mu\psi(\theta\sigma^\mu\bar\theta) &= \theta^a\partial_\mu\psi_a\theta^b\sigma^\mu{}_{b\dot c}\theta^{\dot c} = -\theta^a\theta^b\partial_\mu\psi_a\sigma^\mu{}_{b\dot c}\theta^{\dot c}\\
&= \frac{1}{2}\varepsilon^{ab}(\theta\theta)\partial_\mu\psi_a\sigma^\mu{}_{b\dot c}\theta^{\dot c} = -\frac{1}{2}(\theta\theta)\partial_\mu\psi^b\sigma^\mu{}_{b\dot c}\theta^{\dot c} \qquad (A.12)
\end{aligned}$$

As we mentioned above, $\theta_a, \theta_{\dot a}$ are Grassman numbers. That means, they anticommute among themselves: $\{\theta_a,\theta_b\} = \{\theta_{\dot a},\theta_{\dot b}\} = \{\theta_a,\theta_{\dot b}\}$

Despite the fact that they are discrete objects we can construct a differentional calculus for them. We define the derivatives formally as

$$\begin{array}{ll}
\frac{\partial}{\partial\theta^a}\theta^b := \delta_a^b & \frac{\partial}{\partial\theta_a}\theta_b := \delta_b^a\\
\frac{\partial}{\partial\theta_{\dot a}}\theta_{\dot b} := \delta_{\dot b}^{\dot a} & \frac{\partial}{\partial\theta_{\dot a}}\theta_{\dot b} := \delta_{\dot b}^{\dot a}
\end{array}$$

When applying derivation on the product of Graasman variables one must take into account the anticommutative nature of Grassman numbers

$$\partial_a(\theta^b\theta^c\ldots\theta^d) = \delta_a^b\theta^c\ldots\theta^d - \delta_a^c\theta^b\ldots\theta^d+\ldots$$

By using the metric tensor we can raise and lower indices of derivatives:

$$\varepsilon^{ab}\partial_b = -\partial^a \quad \varepsilon_{ab}\partial^b = -\partial_a$$

Here we present useful relations:

$$\begin{aligned}
\partial_a\theta_b &= -\varepsilon_{ab} & \partial^a\theta^b &= -\varepsilon^{ab}\\
\partial_{\dot a}\theta_{\dot b} &= -\varepsilon_{\dot a\dot b} & \partial^{\dot a}\theta^{\dot b} &= -\varepsilon^{\dot a\dot b} \qquad (A.13)
\end{aligned}$$

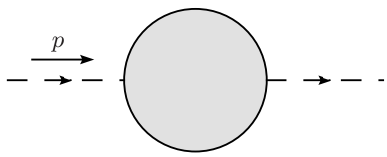
$$\begin{aligned}
\partial_a\theta^2 &= 2\theta_a & \partial_{\dot a}\theta^2 &= -2\theta_{\dot a}\\
\partial^a\theta^2 &= -2\theta^a & \partial^{\dot a}\theta^2 &= 2\theta^{\dot a} \qquad (A.14)
\end{aligned}$$

Appendix B

Generic loop diagrams

B.1 Sfermion self-energies

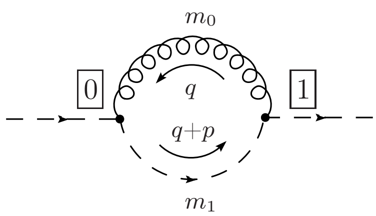
The amplitude for sfermion self-energies is defined as



$$\mathcal{M} = i\,\Pi_{ij}(p^2)$$

where following three diagrams contribute to the $\Pi_{ij}(p^2)$.

The first generic self-energy diagram is one containing vector particle in the loop



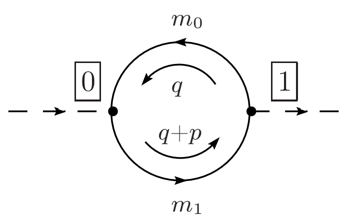
$\boxed{0}$: $ig_0(2p+q)^\mu$

$\boxed{1}$: $ig_1(2p+q)^\nu$

$$\Pi(p^2) = -\frac{g_0 g_1}{4\pi^2}[A_0(m_1^2) + (4p^2 + m_0^2)B_0(p^2, m_0^2, m_1^2) + 4p^2 B_1(p^2, m_0^2, m_1^2)] \tag{B.1}$$

The complete argument in the function $\Pi(p^2)$ is $\Pi(p^2, m_0, m_1, g_0, g_1)$

The second generic diagram includes fermion loop



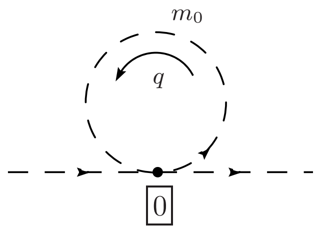
$\boxed{0}$: $i(g_0^L P_L + g_0^R P_R)$

$\boxed{1}$: $i(g_1^L P_L + g_1^R P_R)$

$$
\begin{aligned}
\Pi(p^2) &= -\frac{1}{4\pi^2}[2m_1 m_0(g_0^L g_1^L + g_0^R g_1^R)B_0 + 2(g_0^L g_1^R + g_0^R g_1^L)(p^2 B_1 + p^2 B_{11} + 4B_{00} \\
&+ \frac{1}{6}(p^2 - 3m_0^2 - 3m_1^2))] \\
&= -\frac{1}{4\pi^2}[2m_1 m_0(g_0^L g_1^L + g_0^R g_1^R)B_0 + (g_0^L g_1^R + g_0^R g_1^L)(A_0(m_0^2) + A_0(m_1^2) \\
&+ (m_0^2 + m_1^2 - p^2)B_0)] \qquad \text{(B.2)}
\end{aligned}
$$

where the arguments of the B integrals are the same as in the previous case and the complete argument in $\Pi(p^2)$ is $\Pi(p^2, m_0, m_1, g_0^L, g_0^R, g_1^L, g_1^R)$

The last generic diagram that contributes to the sfermion self-energy is the simplest one

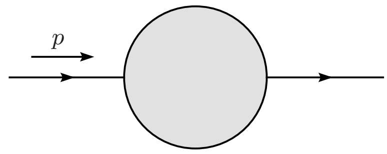


$\boxed{0}$: ig_0

$$\Pi(p^2, m_0) = -\frac{1}{4\pi^2} g_0 A_0(m_0^2) \qquad \text{(B.3)}$$

B.2 Fermion self-energies

The amplitude for fermion self-energies is defined as



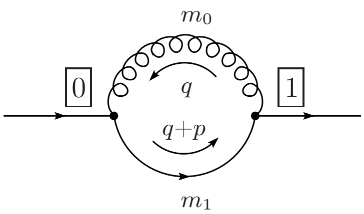
$$\mathcal{M} = i\,\bar{u}(p)\Pi(p^2)u(p)$$

The term $\Pi(p^2)$ further decomposes to

$$\Pi(p^2) = \not{p} P_L \Pi^L(p^2) + \not{p} P_R \Pi^R(p^2) + P_L \Pi^{SL}(p^2) + P_R \Pi^{SR}(p^2) \qquad \text{(B.4)}$$

The following two diagrams contribute to the $\Pi(p^2)$.

The first generic self-energy diagram is one containing vector particle in the loop



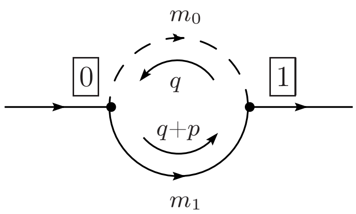
$\boxed{0}$: $i\gamma^\mu(g_0^L P_L + g_0^R P_R)$

$\boxed{1}$: $i\gamma^\mu(g_1^L P_L + g_1^R P_R)$

$$\begin{aligned}
\Pi^L(p^2) &= \frac{1}{4\pi^2} g_0^L g_1^L (2B_0(p^2, m_0^2, m_1^2) + 2B_1(p^2, m_0^2, m_1^2)) && \text{(B.5)}\\
\Pi^{SL}(p^2) &= -\frac{1}{4\pi^2} g_0^L g_1^R m_1 4B_0(p^2, m_0^2, m_1^2) && \text{(B.6)}\\
\Pi^R(p^2) &= \Pi^L(p^2)(L \leftrightarrow R) && \text{(B.7)}\\
\Pi^{SL}(p^2) &= \Pi^{SR}(p^2)(L \leftrightarrow R) && \text{(B.8)}
\end{aligned}$$

The complete argument in all $\Pi(p^2)$ in both diagrams is $\Pi(p^2, m_0, m_1, g_0^L, g_0^R, g_1^L, g_1^R)$

The second generic diagram contains fermion and a scalar particle in the loop



$\boxed{0}$: $i(g_0^L P_L + g_0^R P_R)$

$\boxed{1}$: $i(g_1^L P_L + g_1^R P_R)$

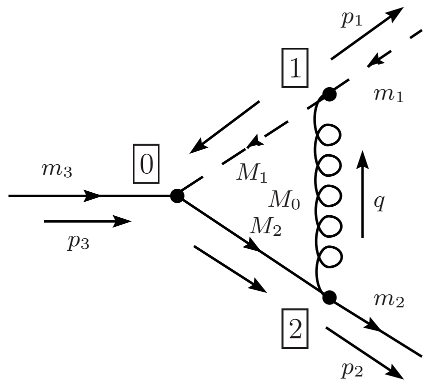
$$\begin{aligned}
\Pi^L(p^2) &= \frac{1}{4\pi^2} g_0^L g_1^R (B_0(p^2, m_0^2, m_1^2) + B_1(p^2, m_0^2, m_1^2)) && \text{(B.9)}\\
\Pi^{SL}(p^2) &= \frac{1}{4\pi^2} g_0^L g_1^L m_1 B_0(p^2, m_0^2, m_1^2) && \text{(B.10)}\\
\Pi^R(p^2) &= \Pi^L(p^2)(L \leftrightarrow R) && \text{(B.11)}\\
\Pi^{SR}(p^2) &= \Pi^{SL}(p^2)(L \leftrightarrow R) && \text{(B.12)}
\end{aligned}$$

B.3 Vertex corrections

The amplitude for the vertex corrections is defined as

$$\mathcal{M} = \frac{i}{4\pi^2} \bar{u}(p_2)(A_L P_L + A_R P_R) u(p_3) \tag{B.13}$$

The first vertex diagram is with the vector particle in the loop



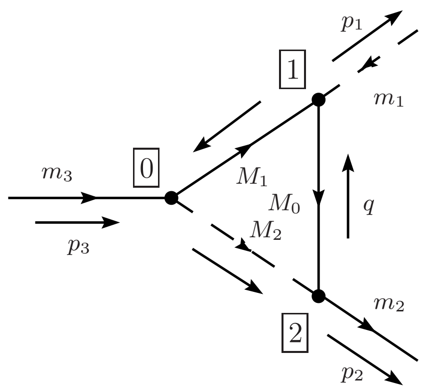
$$\boxed{0}: \quad i(g_0^L P_L + g_0^R P_R)$$
$$\boxed{1}: \quad i g_1 (q - 2p_1)^\mu$$
$$\boxed{2}: \quad i\gamma^\nu (g_2^L P_L + g_2^R P_R)$$

$$\begin{aligned}
A_L^{FSV} &= g_0^L g_1 g_2^R [2C_0(m_1^2 - m_3^2) + C_2(2m_1^2 + m_2^2 - 2m_3^2) + C_1(3m_1^2 - m_3^2) \\
&+ 4C_{00} - \frac{1}{2} + C_{11} m_1^2 + C_{12}(m_1^2 + m_2^2 - m_3^2) + C_{22} m_2^2] \\
&+ g_0^R g_1 g_2^L m_2 m_3 (2C_0 + 2C_2 + C_1) + g_0^L g_1 g_2^L m_2 (2M_2 C_0 + M_2 C_1 + M_2 C_2) \\
&+ m_3 g_0^R g_1 g_2^R (-2M_2 C_0 - M_2 C_1) \qquad \text{(B.14)} \\
A_R^{FSV} &= A_L^{FSV}(L \leftrightarrow R) \qquad \text{(B.15)}
\end{aligned}$$

The argument of A_R^{FSV}, A_L^{FSV} is $A_{R,L}^{FSV}(M_0, M_1, M_2, g_0^L, g_0^R, g_1, g_2^L, g_2^R)$

The argument in all C_i, C_{ij} integrals is $C(m_1^2, m_3^2, m_2^2, M_0^2, M_1^2, M_2^2)$

The second vertex diagram contains two fermions and one scalar in the loop



$$\boxed{0}: \quad i(g_0^L P_L + g_0^R P_R)$$
$$\boxed{1}: \quad i(g_1^L P_L + g_1^R P_R)$$
$$\boxed{2}: \quad i(g_2^L P_L + g_2^R P_R)$$

$$\begin{aligned}
A_L^{FFS} &= -g_0^L g_1^L g_2^L M_0 M_1 C_0 + g_0^L g_1^L g_2^R M_1 m_2 (C_1 + C_2) - g_0^R g_1^R g_2^L M_1 m_3 C_1 \\
&- g_0^L g_1^R g_2^L [m_1^2 C_1 + m_2^2 C_2 + 4C_{00} - \frac{1}{2} + m_1^2 C_{11} + m_2^2 C_{22} + (m_1^2 + m_2^2 - m_3^2) C_{12}] \\
&+ g_0^L g_1^R g_2^R m_2 M_0 (C_0 + C_1 + C_2) - g_0^R g_1^L g_2^L m_3 M_0 (C_0 + C_1) \\
&+ g_0^R g_1^L g_2^R m_2 m_3 C_2 \qquad \text{(B.16)} \\
A_R^{FFS} &= A_L^{FFS}(L \leftrightarrow R) \qquad \text{(B.17)}
\end{aligned}$$

The argument of A_R^{FFS}, A_L^{FFS} is $A_{R,L}^{FFS}(M_0, M_1, M_2, g_0^L, g_0^R, g_1^L, g_1^R, g_2^L, g_2^R)$

The calculation of all presented generic diagrams has been performed in the convention of the LoopTools program [20].

Appendix C

Bremsstrahlung[1]

The following bremsstrahlung integrals are taken from [21]. These integrals are applicable to the processes where a massive particle (p_3, m_3) decays into two massive particles (p_1, m_1) and (p_2, m_2) and a photon (gluon) (q_0, λ). They read as follows

$$I_{i_1,\ldots,i_n}^{j_1,\ldots,j_m} = \frac{1}{\pi^2} \int \frac{d^3p_1}{2p_{10}} \frac{d^3p_2}{2p_{20}} \frac{d^3q}{2q_0} \delta(p_3 - p_1 - p_2 - q) \frac{\pm 2qp_{j_1} \ldots \pm 2qp_{j_m}}{\pm 2qp_{i_1} \ldots \pm 2qp_{i_n}} \tag{C.1}$$

The plus signs belong to p_1, p_2 and the minus sign to p_3.

The decay width for the neutralino decay into antisbottom squark, bottom quark and a gluon reads:

$$\Gamma_{brems} = \frac{1}{2m_3} \frac{g_s^2}{2^5 \pi^3} C_F (cI + c_1 I_1 + c_2 I_2 + c_{11} I_{11} + c_{12} I_{12} + c_{22} I_{22} + c_2^1 I_2^1) \tag{C.2}$$

where the corresponding coefficients are:

$$\begin{aligned}
c &= aa^* + bb^* && \text{(C.3)}\\
c_1 &= 2[(m_1^2 - m_2^2 - m_3^2)(aa^* + bb^*) - 2m_2 m_3 (ab^* + ba^*)] && \text{(C.4)}\\
c_2 &= c_1 && \text{(C.5)}\\
c_{11} &= 2[(m_1^4 - m_1^2 m_2^2 - m_1^2 m_3^2)(aa^* + bb^*) - 2m_1^2 m_2 m_3 (ab^* + ba^*)] && \text{(C.6)}\\
c_{12} &= 2[(m_1^4 - m_2^4 + m_3^4 - 2m_1^2 m_3^2)(aa^* + bb^*) \\
&- 2(m_1^2 m_2 m_3 - m_2 m_3^3 + m_2^3 m_3)(ab^* + ba^*)] && \text{(C.7)}\\
c_{22} &= 2[(-m_2^4 - m_1^2 m_2^2 - m_2^2 m_3^2)(aa^* + bb^*) - 2m_2^3 m_3 (ab^* + ba^*)] && \text{(C.8)}\\
c_2^1 &= aa^* + bb^* && \text{(C.9)}
\end{aligned}$$

where the masses and couplings are: $m_1 = m_{\tilde{f}_i}, m_2 = m_f, m_3 = m_{\tilde{\chi}_k}, a = a_{ik}, b = b_{ik}$. The analytic forms of the bremsstrahlung integrals are given in the mentioned article.

[1] Calculations in the original thesis did not include Bremsstrahlung

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